

## Theoretical & Experimental Shape Optimization Of Cantilever Beam by Constrained Optimization Method

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**Abstract** – The term structural optimization is commonly used for the optimization of engineering structures, such as building, automobile, or airplane structures for improved strength, stiffness properties, fatigue life and reduced weight or cost. Main purpose of this study is to find a practical and proven method to reduce the weight of cantilever beam by using constrained optimization method with respect to desired stress levels. Thus, reducing the weight of components which are similar material and geometry used such as heavy commercial vehicle wheels ventilation holes shows that this approach will be applicable. Additionally, due to reducing weight of heavy commercial vehicle wheels fuel efficiency and cooling performance of brake/hub system increased, if only if desired stress levels are provided. Constrained optimization method is used to ensure that stress levels remain within acceptable limits while optimizing the weight of the cantilever beam. This study conducted in two stages. Firstly, constrained optimization problem is simulated in Altair Hypermesh which is well known computer aided engineering program in literature and industry. Design variable is the hole shape on the cantilever beam. This hole shape constrained to grow or shrink only one planar plane and as a principal stress is restricted 75MPa or less under 9,81N vertical force. On second stage optimized cantilever beam design which is obtained by simulation result is produced. A strain rosette is applied on optimized hole shape critical point. Strain values are measured under same load conditions which are as same as simulation loads. Strain rosette values are converted to principal stress for comparison to simulation results. In this way optimization results are verified by experimental application. Initial design principal stress simulation result is calculated 132,3MPa where this result reduced to 74,6MPa after optimization study. Experimental principal stress measurements are observed 70,2MPa which is around %6 lower than simulation results. And design variable hole weight reduction is observed 11,2gr to 4,7gr which is 2,38 times lighter. Achieved results leads to cost saving on components. Beside cost saving, constrained optimization method may mitigate heavy commercial wheels weight as a first benefit. Due to weight reduction, inertial resistances will decrease and this effect leads to increase fuel efficiency. Additionally, heavy commercial truck vehicle brake/hub systems cooling performance will be positively affected.

**Keywords** – Structural Optimization, Shape Optimization, Strain Measurement, Cantilever Beam Design, Constrained optimization.

### I. INTRODUCTION

Structural optimization is a key element in the functional and technological design of load bearing structures. The engineer is posed with the difficult task of designing a structure by considering objectives that are often times contradictory, like minimizing total mass or volume, minimizing stress, maximizing stiffness, homogenizing stress distribution, ensuring proper manufacturability, minimizing production costs, ect. Structural optimization implies finding the optimum geometry with respect to one or more such criteria.

Structural optimization can be divided in three distinct branches, each targeting different types of parameters: topology, size and shape optimization. The techniques generally target either only topology or only size and shape optimization, with some rare exceptions that try to formulate the problem in a holistic way.

Topology optimization is the most general type of structural optimization, being performed in the initial phases of the

design. All the feasible domain is considered, the aim being to find the most advantageous material distribution inside this domain, with respect to the design objectives. Topology optimization is responsible with most of the objective satisfaction, offering an initial model that can be fine-tuned afterwards with shape and size optimization methods. [1]

Free-shape optimization uses a proprietary optimization technique developed by Altair Engineering, Inc., wherein the outer boundary of a structure is altered to meet with pre-defined objectives and constraints. The essential idea of free-shape optimization, and where it differs from other shape optimization techniques, is that the allowable movement of the outer boundary is automatically determined, thus relieving you of the burden of defining shape perturbations. [2]

Free-shape optimization allows these design grids to move in one of two ways:

- 1- For shell structures, grids move normal to the surface edge in the tangential plane.
- 2- For solid structures, grids move normal to the surface.

Due to limited natural resources and environmental pollution structural optimization studies have become widespread in automotive components. Such as heavy commercial vehicle wheels. This type of wheels should endure required stress levels with respect to minimum design weight. In this study theoretical and experimental shape optimization applied on cantilever beam to find a practical and proven method for weight reduction.

## II. MATERIALS AND METHOD

This study consists mainly 2 stages. Firstly, initial design optimized by shape optimization method as theoretically by using Altair Hyperworks which is finite element analysis software. Afterwards experimental measurements are conducted on shape optimized design for comparison. Guideline of study is given in Fig. 1

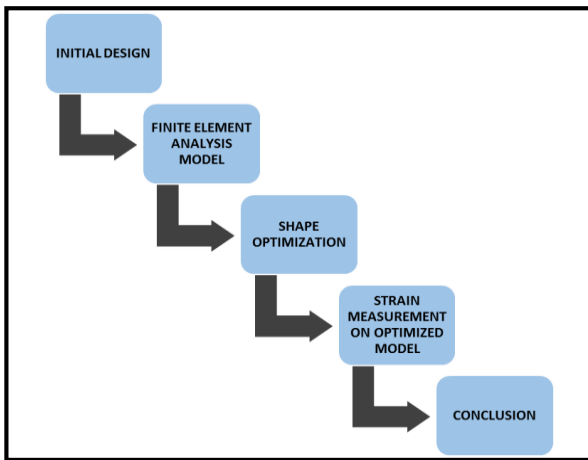


Fig. 1 Guideline of study

### A. Theoretical Shape Optimization

In this stage initial design finite element model of cantilever beam is prepared in Altair Hypermesh.

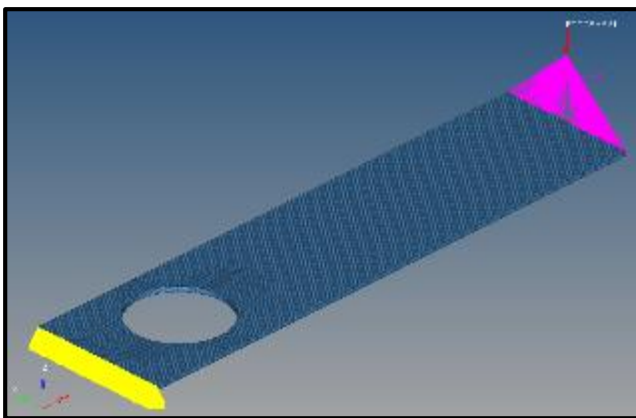


Fig. 2 Initial design

Details of material, geometric values of cantilever beam and finite element model are listed in Table 1.

Table 1. Initial Design Properties

Description	Value
Young Modulus	$E = 210 \text{ GPa}$
Poisson Ratio	$\nu = 0,3$
Density	$\rho = 7,85 \text{ gr/cm}^3$
Applied Force	$F = 9,81 \text{ N}$
Cantilever Length	$l = 200 \text{ mm}$
Rectangular Cross Section	$b = 50 \text{ mm}$ $h = 2 \text{ mm}$
Initial Hole Area	$961,625\text{E-}6 \text{ mm}^2$
Mesh Node Qty	49.980
Elements Qty	38.784
RBE2 Elements Qty	266
SPC Nodes Qty	266

Shape optimization parameters are defined on initial design finite element model. 500 Nodes of hole shape in cantilever beam are selected as a design variable. These nodes are able to perturbate to find desired optimization parameters during optimization iterations. Hole shape design variables are allowed only to grow or shrink on XY plane. This type of constraints is known in literature as manufacturing constraints.

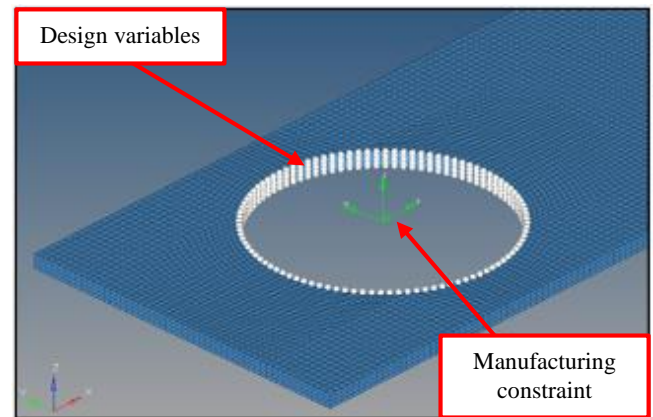


Fig. 3 Design variables and manufacturing constraint

Maximum principal stress (P1) is constrained as not to exceed 75 MPa in all elements of cantilever beam.

Table 2. Optimization parameters

Description	Value
Design Variable	Hole shape
Constraint	Maximum principal stress < 75 MPa
Side Constraint	Hole shape grow/shrink direction = XY Plane
Objective	Minimize mass

Shape optimized finite element model is exported solver deck as .fem file format. Altair OptiStruct is a solver that can solve linear and nonlinear finite element analysis problems. Optimization has converged by OptiStruct after 20 iterations around 12 minutes. Converged shape is given in Fig. 4

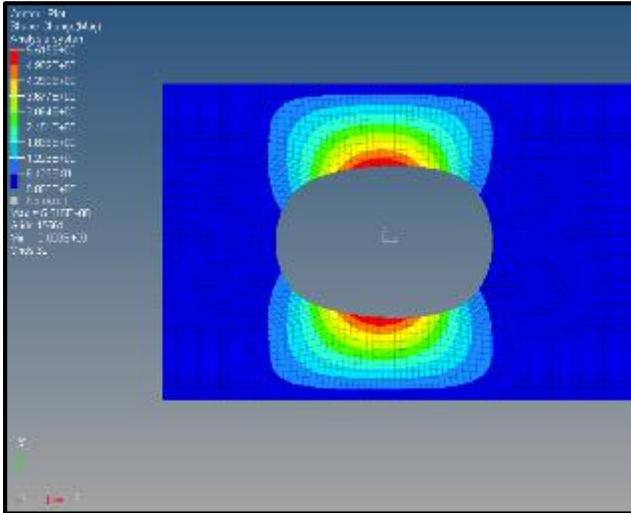


Fig. 4 Optimized hole shape

### B. Experimental Shape Optimization

In many cases theoretical studies are not supported with experimental results. Finding a proven method for structural optimization is highly important. Thus, shape optimized model is manufactured for validation. This stage explains how to experimental study conducted of shape optimization.

The amount of deformation a material experiences due to an applied force is called strain. Strain is defined as the ratio of the change in length of a material to the original, unaffected length, as shown in Fig. 5 Strain can be positive (tensile), due to elongation, or negative (compressive), due to contraction. When a material is compressed in one direction, the tendency to expand in the other two directions perpendicular to this force is known as the Poisson effect. Poisson's ratio ( $\nu$ ), is the measure of this effect and is defined as the negative ratio of strain in the transverse direction to the strain in the axial direction. Although dimensionless, strain is sometimes expressed in units such as in./in. or mm/mm. In practice, the magnitude of measured strain is very small, so it is often expressed as microstrain ( $\mu\epsilon$ ), which is  $\epsilon \times 10^{-6}$ . [3]

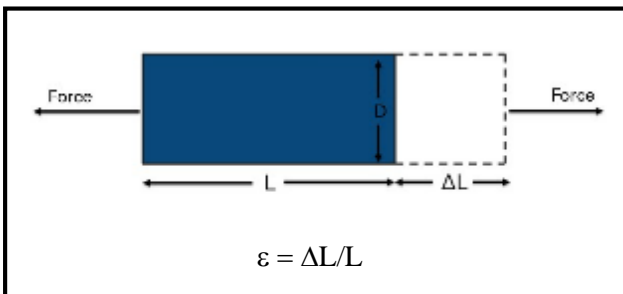


Fig. 5 Strain

A strain gage can measure strain in only one direction the axis along which the strain gage is mounted. To effectively measure the three independent components of plane strain (extensional strain along X and Y axis, as well as shear strain), three independent strain measurements are needed. Strain gage rosettes are used to perform such measurements.

A strain gage rosette is an arrangement of two or three closely positioned strain gages, separately oriented to measure the strains along different directions of the underlying surface of the object being measured.

Strain-gage manufacturers offer three basic types of strain gage rosettes.

- Tee Rosette—A tee rosette consists of two gages oriented at 90 degrees with respect to each other.
- Rectangular Rosette—A rectangular rosette consists of three strain gages, each separated by a 45° angle.
- Delta Rosette—A delta rosette consists of three strain gages, each separated by a 60° angle.

Generally, if the direction of principal stress is uncertain in structure stress measurement, a triaxial rosette gage is used and measured strain values are calculated in the following equation to find the direction of the principal stress. [4]

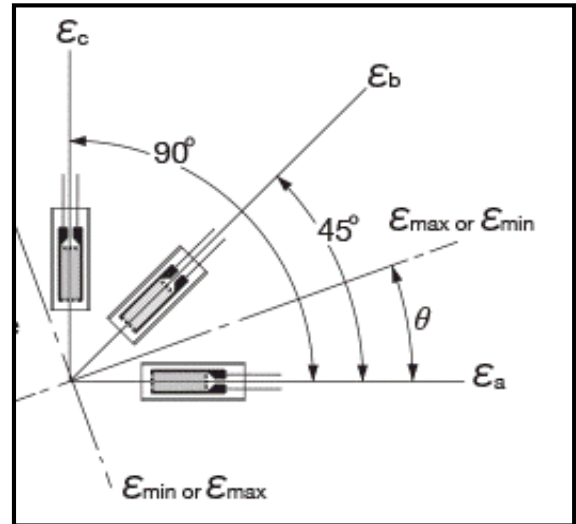


Fig. 6 Strain rosette

$$\sigma_{max} = \frac{E}{2(1-\nu^2)} \left[ \begin{array}{l} \text{Max. principal stress} \\ (1+\nu)(\epsilon_a + \epsilon_c) + \\ (1-\nu) \times \sqrt{2\{(\epsilon_a - \epsilon_b)^2 + (\epsilon_b - \epsilon_c)^2\}} \end{array} \right]$$

$$\sigma_{min} = \frac{E}{2(1-\nu^2)} \left[ \begin{array}{l} \text{Min. principal stress} \\ (1+\nu)(\epsilon_a + \epsilon_c) - \\ (1-\nu) \times \sqrt{2\{(\epsilon_a - \epsilon_b)^2 + (\epsilon_b - \epsilon_c)^2\}} \end{array} \right]$$

In this study HBM 1-RY9x-3/350 rectangular rosette is applied on optimized cantilever beam maximum principal stress area.



Fig. 7 Strain rosette on shape optimized model

### III. RESULTS

Heretofore initial design is created as a finite element analysis model. Shape optimization application is conducted on initial design with respected to desired load conditions and constraints. Optimized design is manufactured and strain rosette is applied on maximum principal stress area. Maximum principal stress results of initial design, traditional design, shape optimized design and experimental study are given as below.

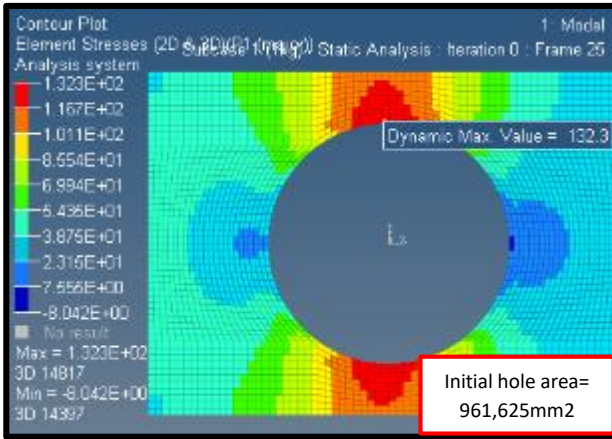


Fig. 8 Initial design theoretical result

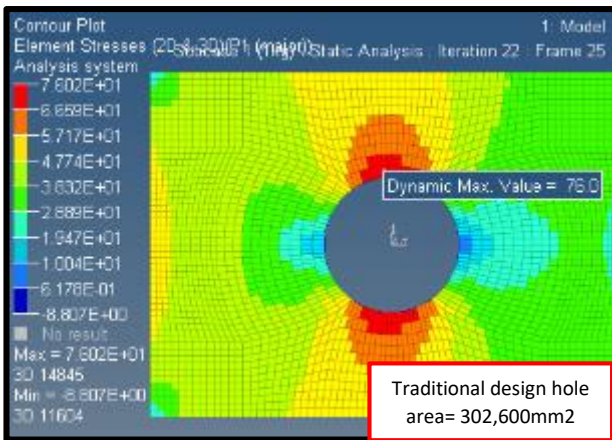


Fig. 9 Traditional design theoretical result

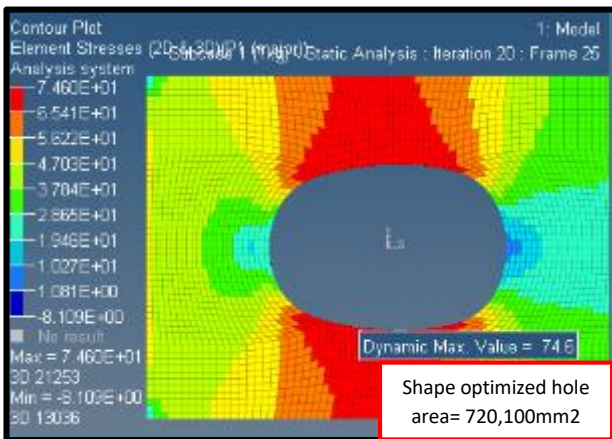


Fig. 10 Shape optimized design theoretical result

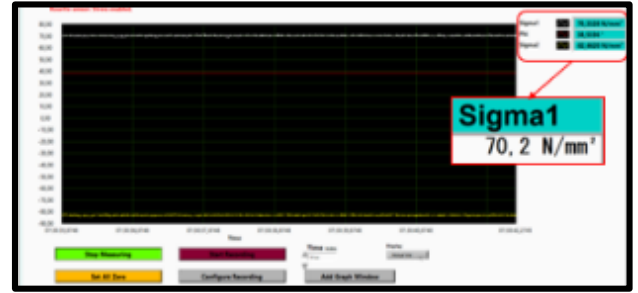


Fig. 11 Experimental results

### IV. DISCUSSION

So far theoretical and experimental results obtained from the simulation and experiments are presented. Summary of optimization studies results are given in Table 3.

Table 3. Summary of optimization studies

Design	Maximum principal stress [MPa]	Surface [mm <sup>2</sup> ]	Hole shape weight [gr]
Initial	132,3	961,625	15,1
Traditional	76,0	302,600	4,7
Shape optimized	74,6	720,100	11,2
Experimental	70,2	~720,100	~11,2

According results it is observed that shape optimized hole design is around 2,38 times lighter than traditional design with respected to similar stress levels.

Approximately %6,3 maximum principal stress deviation observed between finite element analysis and experimental results. It is known that in every measurement results some deviation will be occurred. In this case %6,3 deviation comes from application of strain rosette position. In Fig 13. Elements which are near to maximum principal stress area are showed.

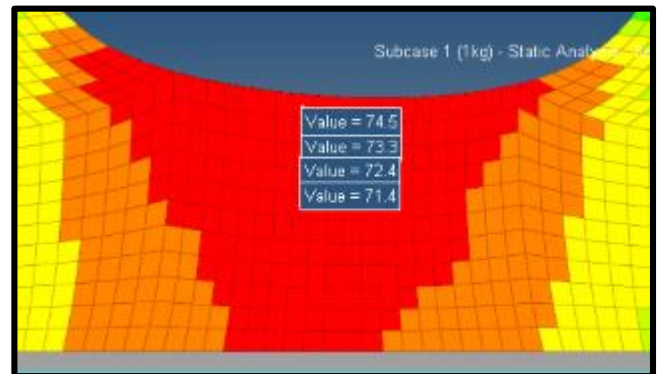


Fig. 12 Maximum principal stress results

The maximum principal stress at the exact position of strain rosette is between 72,4MPa and 74,5MPa. It shows total deviation is between %3 - %6,3.

### V. CONCLUSION

For carmakers, reducing the cost and weight of their vehicles is a constant challenge. While many organizations take action at the production stage – either by using cheaper versions of the same vehicle parts and systems or by improving efficiency across the supply chain – there are more substantial

savings to be made by optimizing cost and weight while the technical concept is still under development. [5]

Optimization can be viewed as a process that searches methodically for better answers, better solutions, or better designs that a human being may not be able to find through experience, intuition, or courageous trial and error. Optimization can be defined as the art of making things better. Interestingly, optimization very often does not simply allow us to do something better, but it may also make it possible to do something that we did not otherwise know how to do. [6]

Shape optimization of a cantilever beam is a pilot scheme for weight reduction of heavy commercial vehicle wheels. Presented study shows us that shape optimization method is proven method for weight reduction studies. It is thought that for future studies shape optimization method applicable for heavy commercial vehicle wheels.

#### ACKNOWLEDGMENT

This research was supported by Maxion Wheels Jantaş (JAN) Inc. for increasing R&D abilities. Finite element analysis is done with JAN licensed Altair HyperWorks 2017 program. All data acquisition system is supported by JAN. For data acquisition system BMC Messsysteme GmbH USB-AD16f DAQ is used. Measurements and evaluation studies are done with JAN licensed LabVIEW 2015 programme.

We would like to thanks for supporting, sharing software and hardware to Maxion Wheels Jantaş Inc.

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