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Pink Noise Generation from White Noise using Current Mode Analog Building Blocks

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Abstract – Pink noise, which has a power spectral density that is inversely proportional to the frequency, is widely used for acoustic applications such as for testing audio equipment. Its generation from white noise requires such a system transfer function that it is not directly realizable in the analog domain, and it is necessary to use an approximation for the frequency of interest. In this study, it is generated from white noise using a fourth-order filter transfer function. The presented circuit is different from the conventional voltage mode pink noise generators in the literature. It uses operational transconductance amplifiers (OTAs) as active building blocks for the implementation and operates in the current mode. Using transconductances for filter matching conditions enables to design circuits that do not require large capacitance and resistance values, and this is generally the preferred case when integrated circuit applications are considered. The active realization in this paper is verified via SPICE simulations. The simulations agree well with the theory, and it can be claimed that adopting current mode approach to analog pink noise generators, especially when integrated circuit applications are regarded, The OTA circuit is a feasible alternative to conventional ones employing operational amplifiers.

Keywords – Pink noise, analog filters, approximation, signal processing, current mode

I. INTRODUCTION

Normally, noise is not desired in electronic systems due to its unpredicted and disruptive behavior which usually limits the performance and undermine the efficiency of the overall system [1]. Furthermore, it is one of the causes of reliability problems in circuits and affect the operation of systems comprising transistors severely [2],[3]. Although its instantaneous power is unpredictable, on average, noise can be categorized into groups according to the shape of its power spectral density (PSD). These categories are named as colors such as white, pink, blue, brown etc. White noise is probably the most traditionally reviewed noise type, and it has a uniform spectral density whereas pink noise has a power spectral density that is inversely proportional to frequency.

In some applications, using some form of noise for a specific purpose might be beneficial. For example, pink noise can be used for measuring the characteristics of electroacoustic systems or calibrating audio equipment where the bandwidths of test bands are linked by a constant factor of 1/10 of a decade for the pink noise case. In addition, white noise, sinusoidal signals or signals similar to impulse function can also be used in the testing of those systems [4].

Pink noise is generally mixed with other forms of noise. Therefore, it is generally not an easy task to obtain it directly. To solve this problem, one of the approaches is to use white noise and pass it through a system or a device that accepts this white noise at the input and transforms it into pink noise at the output by means of changing its frequency spectrum[5]. However, using conventional analog filtering methods, this is not a viable choice due to the frequency characteristics of pink noise. For a system that generates pink noise from white noise, there is 10dB of change per decade whereas even for a first order analog filter, the roll-off is 20dB per decade. In the literature, the approach of fractional order integrators or similarly, the approximations to the actual transfer function are utilized in order to realize the pink noise generators. In those circuits, however, voltage mode circuits are mainly used for the implementations and those choices lead to circuits having large passive elements [4],[5].

In this study, applying current mode approach to the problem and using the circuits employing operational transconductance amplifiers, it is shown that the implementation of pink noise generation is possible with using filtering circuits having small passive element values. That is an important requirement when integrated circuit solutions are necessary.

II. THE APPROXIMATE TRANSFER FUNCTION FOR PINK NOISE GENERATION

The transfer function for a system that gives pink noise output from white noise input, which means that the filter has a frequency response as 10dB per decade roll-off, is given in (1). In the equation, H is a coefficient that can be determined according to the normalization procedure. As it is seen from the equation, this is not a realizable transfer function by using standard, nonfractional analog filtering methods because of the existence of the noninteger power of the complex frequency variable in the denominator polynomial.

$$T(s) = \frac{H}{\sqrt{s}} \tag{1}$$

The equation in (1), considering the magnitude, can be derived using PSD of white noise and pink noise as shown in the following equations. $\mathrm{PSD_W}(f)$ and $\mathrm{PSD_P}(f)$ represents the power spectral densities of white noise and pink noise respectively.

$$\mathrm{PSD}_W(f) = \mathrm{N}_W \tag{2}$$

$$\mathrm{PSD}_P(f) = \mathrm{N}_P/f \tag{3}$$

To transform white noise into pink noise and when the input and the output of the system are considered, for an LTI system, the corresponding relation is

$$\mathrm{PSD}_P(f) = |T(f)|^2\mathrm{PSD}_W(f) \tag{3}$$

which gives

$$|T(f)| = \frac{\sqrt{N_P/N_W}}{\sqrt{f}} \tag{4}$$

It is possible to approximate the actual transfer function in (1) by using singularities, that are the poles and zeros of the approximating function, in a specific order with a k factor such as

$$\begin{gathered} \mathrm{z}_n = (k)\mathrm{p}_n\,,\ \mathrm{p}_n = (k)\mathrm{z}_{n-1},\ \mathrm{z}_{n-1} = (k)\mathrm{p}_{n-1}, \\ \mathrm{p}_{n-1} = (k)\,\mathrm{z}_{n-2}\ \ldots,\ \mathrm{z}_1 = (k)\,\mathrm{p}_1 \end{gathered} \tag{5}$$

Using the method above, zeros and poles of the fourth-order approximation are obtained, and the results are tabulated in Table 1. The singularities were determined according to the k factor of 2.882. That value is the optimum value that minimizes the Mean Square Error (MSE) for that fourth-order approximation [5]. After necessary normalizations, following transfer function is finally reached.

$$H(S) = \frac{2.54s^4 + 117.28s^3 + 581.21s^2 + 337.96s + 21.09}{8.31s^4 + 133.15s^3 + 228.95s^2 + 46.19s + 1}$$
(6)

Fig. 1 shows the ideal and approximated magnitude functions where for the ideal function, H=2.51 is used in SPICE for the normalization to achieve 0 dB gain at 1Hz.

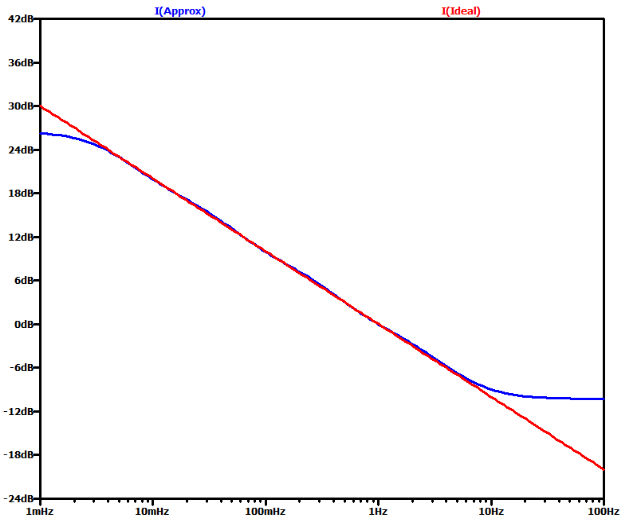

Fig. 1 Ideal and approximated magnitude responses

Next figure shows the phase response of the ideal and the approximated functions. Although the approximation performs well for the magnitude response, this cannot be told likewise for the phase response.

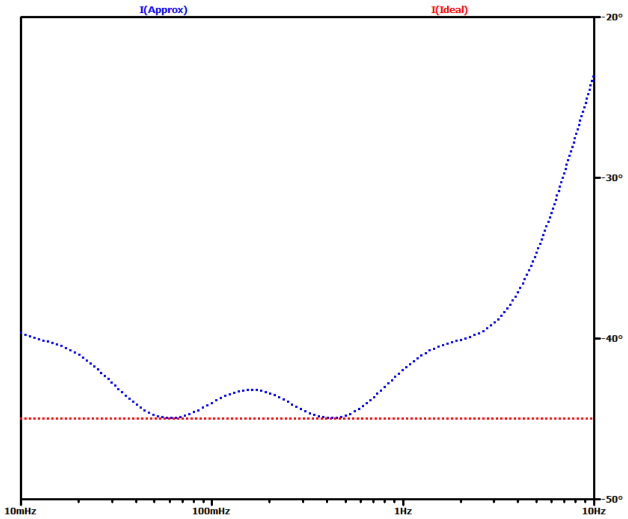

Fig. 2 Ideal and approximated phase responses

The phase of the approximated transfer function becomes close to the ideal one only for two narrow frequency regions. A better approximation method that performs well for both phase and magnitude would be a fine addition to the available literature. One possible solution might be the utilization of some sort of allpass filter blocks cascaded to the output of the circuit realizing the approximated transfer function. This method enables to adjust the phase without changing the magnitude response that has already been optimized.

Table 1. Singularities of the approximated transfer function

Zeros (rad/s)	**Poles (rad/s)**
281.80	97.78
2340.63	812.15
19441.06	6745.68
161475.96	56029.13

III. THE OTA CIRCUIT

The implementation of the transfer function in (6) has been accomplished by using current mode approach and utilizing dual output OTA blocks. The circuit is shown in Fig. 3. The design is based on multiple loop feedback, leap-frog OTA-C filters [6]. The required equations for the matching conditions and the explicit design formulas can be found in [7]. The fourth order circuit in Fig. 3 requires only four capacitors and OTA transconductances are used instead of resistors. OTA-a1 – OTA-a5 enable the realization of the numerator polynomial where OTA1–OTA4 are used for the denominator polynomial. OTA-r behaves like a resistor and makes current to voltage conversion which is necessary for the intermediate nodes. In SPICE simulations, behavioral models of OTAs are used. The transconductance and capacitor values for the pink noise generator are given in Table 2. The negative transconductance values can be easily realized by interchanging the inputs. The SPICE simulation results for magnitude and phase responses are given in Fig. 4 and Fig.5 respectively.

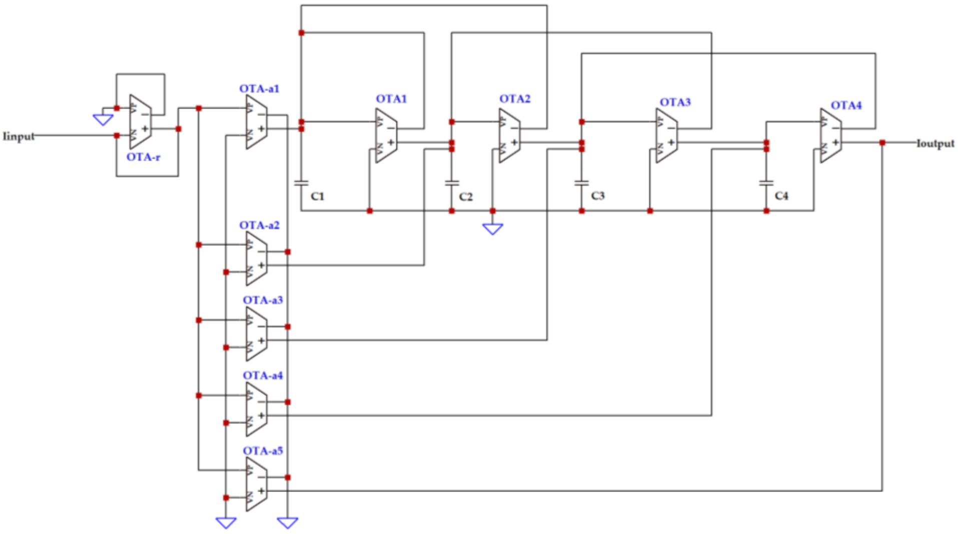

Fig. 3 OTA-based pink noise generator from white noise

The results in Fig 4 and Fig. 5 are acquired after frequency scaling the transfer function in (6) to the ranges that are close to audio frequencies by using (s/1000) in place of s.

Table 2. Component values

Capacitors	Transconductances of OTAs
C1 = 6.2398 pF C2 = 5.8898 pF C3 = 4.9571 pF C4 = 4.5605 pF	OTA-r =1 nA/V OTA1 = 0.1 µA/V OTA2 = 0.01 µA/V OTA3 = 1 nA/V OTA4 = 0.1 nA/V OTA-a1 = -136.2205 µA/V OTA-a2 = 155.307 µA/V OTA-a3 = -19.4867 µA/V OTA-a4 = 0.4204 µA/V OTA-a5 = 0.3056 nA/V

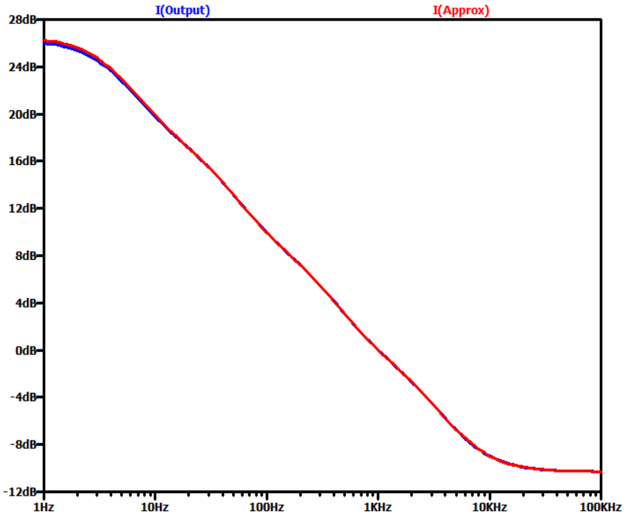

Fig. 4 Circuit output and the approximation magnitude responses

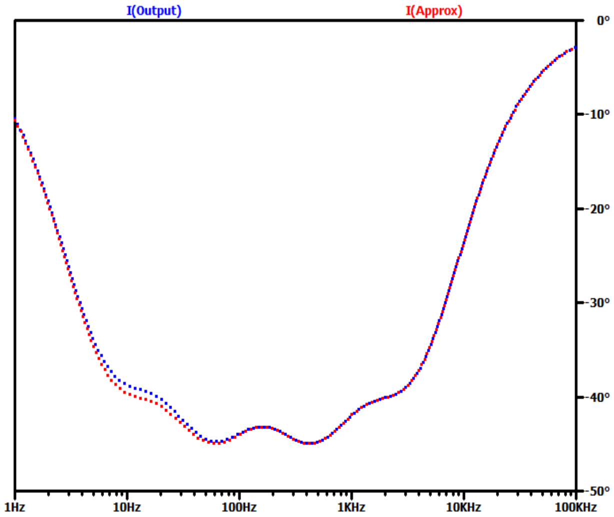

Fig. 5 Circuit output and the approximation phase responses

As it is seen from the figures, dual output OTA based design is properly simulating the approximated function. Small deviations are caused by rounding and numerical accuracy errors.

IV. CONCLUSION

In this study, generation of pink noise from white noise has been discussed. The ideal and the approximated transfer functions with related singularities have been given. The necessary relations for the poles and zeros of the approximation have been shown. For the implementation, an OTA based current mode circuit is designed for audio frequencies. Using this method, by properly adjusting the transconductance values of OTAs, a final circuit has been reached comprising only four capacitors with the values in the range of a few picofarads. Further reduction of these values is also possible by adjusting transconductances if necessary. SPICE results for the OTA circuit and the approximation have been given. The theoretical study agrees well with the simulations.

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