

Effect of Non-Gaussian Noise on Detection Performance of Multi-Stage Detectors

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Abstract – It is well known that detection performances of conventional detection methods deteriorate under non-Gauss noise. This problem is also encountered in cognitive radio where impulsive nature of noise negatively affects spectrum sensing. Multistage detection techniques can be considered as a possible solution as they analyse signal samples using different detection techniques in a sequential manner. In this study, spectrum sensing performance of a multistage detection method is investigated for detection of a dynamic primary user signal contaminated by non-Gaussian noise. At first, observed signal samples are analysed by energy detector and an ensuing absolute value cumulation computation. Then, a decision as to whether the channel is in use or not is given based on the result of this analysis. Accordingly, if any one of those techniques detects the primary user signal, then the channel is assigned as busy.

Keywords – Signal detection, Multi-stage detection, Energy detector, Absolute value cumulation, Spectrum sensing, Cognitive radio.

I. INTRODUCTION

Ever increasing number of wireless devices makes it mandatory to use the electromagnetic spectrum in the most efficient way. Cognitive radio (CR) is proposed by Joseph Mitola III in 1999 as a promising solution of this problem of electromagnetic spectrum scarcity [1, 2].

CR users, which are also named as secondary users (SUs), periodically scan the frequency spectrum and communicate over idle channels. In this way, frequency bands currently not used by licensed (or primary) users are determined and SUs utilize those channels for their own communication needs. Therefore, accurate sensing of the current status of the channel is highly crucial to prevent occurrence of any interference among primary users (PUs) and SUs [3]-[5].

In the cognitive radio literature, noise is generally assumed Gaussian distributed. However, there are still a number of studies in which noise is modelled as non-Gaussian. It is well known that non-Gaussian noise negatively affects the detection performance of conventional detection techniques [6]-[8]. Another factor, which deteriorates detection performance, is the existence of a dynamic PU in the channel [9]-[15]. It is shown in [16] that simultaneous existence of a dynamic PU and non-Gaussian noise in the channel dramatically decreases the detection performance of the energy detector.

In this study, a multi-stage spectrum sensing scheme is proposed for detection of a dynamic PU in non-Gaussian noise. Multi-stage sensing methods are used in the CR literature particularly for designing robust signal detectors [17, 18]. In our proposed scheme, observed signal samples are firstly processed by energy detector (ED) [3] followed by the

application of the absolute value cumulation (AVC) method [19, 20]. In the end, a decision is taken based on the result of this two-stage analysis.

The rest of the manuscript is organized as follows: The signal model is presented in Section II. The proposed multi-stage sensing scheme is introduced in Section III. Simulation results are given in Section IV, and finally, our conclusions are presented in Section V.

II. SIGNAL MODEL

Spectrum sensing in CR is generally formulated as a binary hypothesis problem [3]. Accordingly, \mathcal{H}_0 and \mathcal{H}_1 denote the sensing states corresponding to absence and presence of the signal, respectively. That is; \mathcal{H}_0 is the null hypothesis indicating that PU is not communicating, and \mathcal{H}_1 is the alternative hypothesis indicating the existence of the PU in the channel.

It is possible to define four distinct cases as listed below:

1. Declaring \mathcal{H}_1 under \mathcal{H}_1 hypothesis which is called probability of detection (P_D),
2. Declaring \mathcal{H}_0 under \mathcal{H}_1 hypothesis which corresponds to probability of miss (P_M),
3. Declaring \mathcal{H}_1 under \mathcal{H}_0 hypothesis which is named as probability of false alarm (P_{FA}),
4. Declaring \mathcal{H}_0 under \mathcal{H}_0 hypothesis.

PU may be static or dynamic during the sensing period; i.e. PU status might change within the sensing period. Assuming first that the PU is static, binary hypothesis testing problem is given as follows:

$$\begin{aligned} \mathcal{H}_0: x[n] &= w[n], \quad n = 1, 2, 3, \dots, N \\ \mathcal{H}_1: x[n] &= s[n] + w[n], \quad n = 1, 2, 3, \dots, N \end{aligned} \quad (1)$$

where $x[n]$ is the received signal, $s[n]$ is the real-valued PU signal, and $w[n]$ denotes noise samples. N is the total number of observed signal samples.

In this study, we assume that noise is zero-mean Laplacian with the variance of σ^2 . As a specialized form of generalized Gaussian distribution (GGD), Laplacian noise is frequently employed in the literature for modelling impulsive noise [19]-[23]. Accordingly, GGD is given as

$$f_X(x) = \frac{c_1(\beta)}{\sqrt{\sigma^2}} \exp\left(-c_2(\beta) \left|\frac{x}{\sqrt{\sigma^2}}\right|^{\frac{2}{1+\beta}}\right) \quad (2)$$

where $\beta > -1$, and

$$c_1(\beta) = \frac{\Gamma\left[\frac{3}{2}(1+\beta)\right]}{(1+\beta)\Gamma\left[\frac{3}{2}(1+\beta)\right]}, \quad (3)$$

$$c_2(\beta) = \left[\frac{(1+\beta)}{\frac{1}{2}(1+\beta)}\right]^{\frac{1}{1+\beta}}. \quad (4)$$

In Eqns. (3) and (4), $\Gamma(\cdot)$ denotes the gamma function which is defined as

$$\Gamma(x) = \int_0^x u^{x-1} e^{-u} du. \quad (5)$$

Gaussian probability density function (pdf) is obtained by assigning $\beta = 0$ in Eq. (2). If parameter β is taken as $\beta = 1$, then the pdf of Laplacian distribution is obtained as [24]

$$f_L(x) = \frac{1}{\sqrt{2\sigma^2}} \exp\left(-\sqrt{\frac{2}{\sigma^2}} |x|\right). \quad (6)$$

For different σ^2 values, the shape of the pdf is shown in Fig. 1. In the figure, the standard normal (Gaussian) distribution $\mathcal{N}(0,1)$ has also been simulated to emphasize tail characteristics of Laplace distribution. Accordingly, probability of obtaining outliers is higher for Laplace pdf as its tails have larger values compared to Gaussian pdf. In addition, higher σ^2 values increase probability of occurrence for outliers in Laplace distribution.

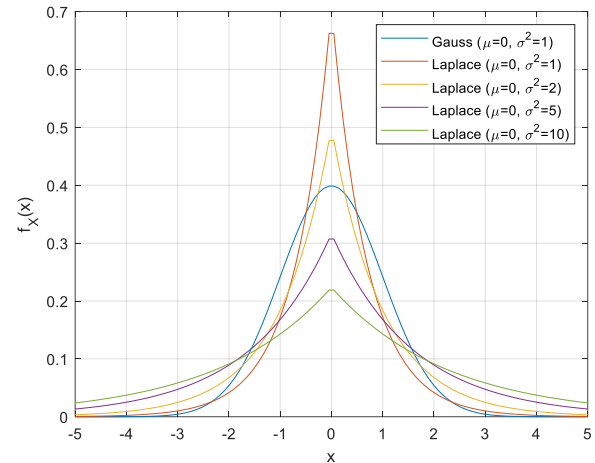


Figure 1. Comparison of Laplacian distributions having different σ^2 values with the standard normal (Gaussian) distribution.

In this study, dynamic PU scenario has also been realized by changing the PU status within the sensing period. This scenario can be mathematically represented as follows:

$$\begin{aligned} \mathcal{H}_0: x[n] &= s[n] + w[n], \quad n = 1, 2, 3, \dots, J_0 \\ &= w[n], \quad n = J_0 + 1, \dots, N \\ \mathcal{H}_1: x[n] &= w[n], \quad n = 1, 2, 3, \dots, J_1 \\ &= s[n] + w[n], \quad n = J_1 + 1, \dots, N \end{aligned} \quad (7)$$

where J_0 and J_1 are departure and arrival indices of the PU. We would like to note that J_0 and J_1 are randomly assigned using Poisson distribution. Accordingly, arrival and departure times of the PU can be generated by

$$J_0 = -\frac{1}{\lambda_d} \ln(1 - u) \quad (8)$$

$$J_1 = -\frac{1}{\lambda_a} \ln(1 - u) \quad (9)$$

where λ_d , λ_a and u are departure rate, arrival rate and uniform random variable with $u \sim U(0,1)$, respectively.

In the next section, proposed sensing scheme is introduced.

III. PROPOSED SENSING SCHEME

In this section, our proposed multi-stage-sensing scheme is introduced. Accordingly, the observed data samples are sequentially analysed by ED and AVC techniques.

Sensing procedure is illustrated as a flowchart in Fig. 2.

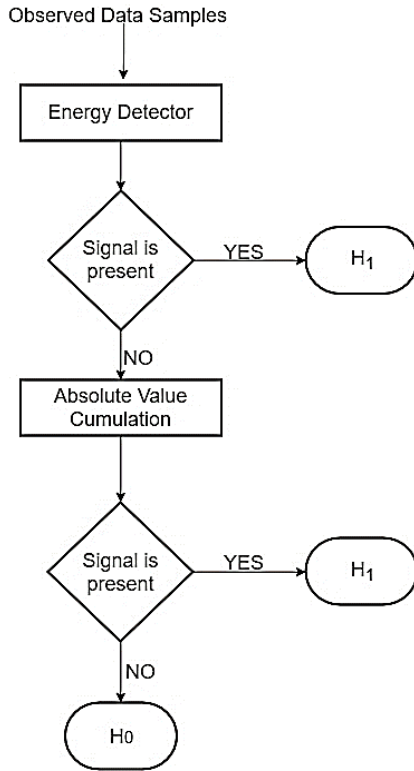


Figure 2. Proposed ED & AVC based multi-stage sensing scheme

According to Fig. 2, observed data samples are firstly applied to ED. The hypothesis test for ED is given as

$$T_{ED} = \sum_{n=1}^N x^2[n] \begin{cases} \mathcal{H}_1 & > \gamma_{ED} \\ \mathcal{H}_0 & < \gamma_{ED} \end{cases} \quad (10)$$

In Eq. (10), γ_{ED} is calculated as [23]

$$\gamma_{ED} = Q^{-1}(P_{FA})2\sqrt{5N}b^2 + 2Nb^2 \quad (11)$$

where N is the total number of observed data samples,

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{+\infty} e^{-\frac{v^2}{2}} dv \text{ and } b = \sqrt{\frac{\sigma^2}{2}}.$$

The algorithm is terminated at this step if ED decides \mathcal{H}_1 hypothesis is true. If \mathcal{H}_0 is decided, then the observed data samples are applied to AVC and the following hypothesis test is carried out

$$T_{AVC} = \sum_{n=1}^N |x[n]| \begin{cases} \mathcal{H}_1 & > \gamma_{AVC} \\ \mathcal{H}_0 & < \gamma_{AVC} \end{cases} \quad (12)$$

where γ_{AVC} is the detection threshold of AVC given as [23]

$$\gamma_{AVC} = Q^{-1}(P_{FA})\sqrt{5N}b + Nb. \quad (13)$$

In the proposed sensing scheme, the final decision is taken by AVC detector. If it decides \mathcal{H}_1 is true, the channel is assigned as busy, and idle if it decides \mathcal{H}_0 . Simulation results

obtained using this sensing procedure are given in the next section.

IV. SIMULATION RESULTS

In this section, simulation results performed to determine the performance of the proposed sensing scheme are presented. In Figs. 3 and 4, the total number of the observed data samples is $N = 200$ and 10^5 realizations have been run in Monte Carlo simulations. The solid and dashed lines in the figures represent the results for static and dynamic PU, respectively. Arrival and departure rates of the dynamic PU have been assigned as $\lambda_a = \lambda_d = 1$. When generating both figures, the PU signal has been modelled as $\mathcal{N}(0,1)$. Noise variances in Figs. 3 and 4 are taken as $\sigma_1 = 1$ and $\sigma_2 = 2$ corresponding to 0 and -3 dB GSNR (generalized signal-to-noise ratio) levels, respectively, where GSNR is defined as $GSNR = 10 \log_{10}(\sigma_s^2/\sigma_w^2)$. For comparison, energy detection and AVC methods are simulated individually in addition to the proposed sensing technique.

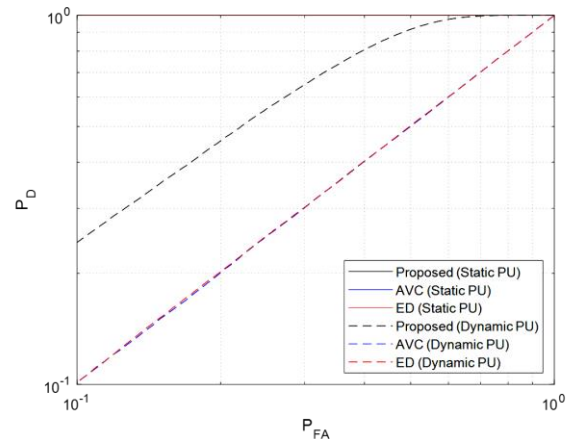


Figure 3. P_D vs. P_{FA} for GSNR = 0 dB.

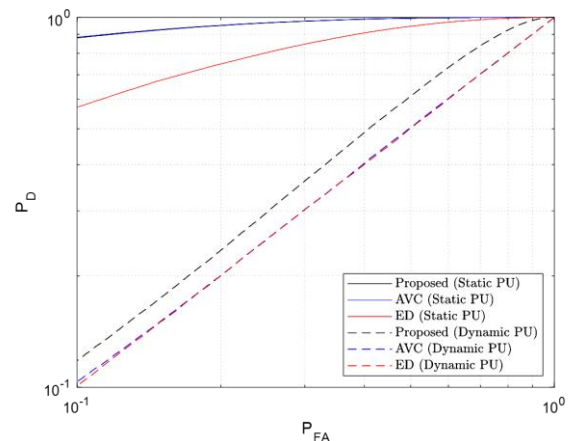


Figure 4. P_D vs. P_{FA} for GSNR = -3 dB.

According to the obtained results, it can be said that the proposed method outperforms other techniques in terms of probability of detection when the PU is dynamic. It is intuitive to observe a performance degradation in Fig. 4 compared to Fig. 3 since detection performance is decreased for all considered methods under lower GSNRs. On the other hand, detection performances of AVC and the proposed technique

are nearly equal. They have higher P_D values than ED when the PU is assumed to be static during the sensing period.

V. CONCLUSIONS

In this study, a multi-stage spectrum sensing scheme is proposed for detection of dynamic PUs contaminated by Laplacian noise. Accordingly, the observed data samples are applied sequentially to ED and AVC. Firstly, signal samples are processed by the energy detector and then, they are analysed by AVC. Finally, a decision is taken based on the result of this analysis. If either ED or AVC manages to detect the PU, then the channel is assigned as busy. Simulations have been carried out for both static and dynamic PU scenarios. The proposed sensing scheme provides a significant performance enhancement in terms of probability of detection although its contribution is limited for static PU case. It is considered that the proposed strategy will pave the way for other studies to investigate multi-stage sensing strategies for detection of dynamic PUs under non-Gaussian noise.

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