

## Statistical precision of Malmquist Productivity Index and DEA: A bootstrap application to OECD healthcare

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**Abstract** – Data Envelopment Analysis (DEA) is a non-parametric mathematical programming that uses multi inputs to produce multi outputs. It is investigated whether the decision-making unit is effective or not. However, it has been debated for years that the results obtained from DEA do not provide statistical inference. Simar and Wilson (1998) develop a bootstrap procedure which may be used to estimate confidence intervals for distance functions used to measure technical efficiency, and demonstrate that the key to statistically consistent estimation of these confidence intervals lies in the replication of the unobserved data-generating process. This paper examines to the case of Malmquist indices constructed from nonparametric distance function estimates using data of OECD countries during the period of 2010 through 2014. We used DEA to derive efficiency scores, Malmquist indices to assess productivity growth, second-stage bootstrapping to determine the accuracy of estimators and whether conclusions would change after considering this statistical information. According to the study, due to the widespread return among countries, the most appropriate scale of operational health services should be investigated instead of treatment centered on people and focused on disease prevention.

**Keywords** – Efficiency, Malmquist DEA, Bootstrap, Healthcare

### I. INTRODUCTION

The process of globalization of health care has increased substantially in recent years, expedited by several broad factors [2]. A key policy challenge in most OECD countries is to improve outcomes of the health care system while containing its costs. Benchmarking countries and identifying best practices to enhance public spending cost-effectiveness would, in this regard, be a useful exercise [1]. It is not always possible to assign countries to specific models and speak of a clear system classification, as countries often do not follow a single financing model and may change policy over time. Health policies in OECD member countries are country specific and health services are organized at national level. Therefore, there are profound differences between countries in terms of health financing, service provision, manpower practices and health legislation. When health care financing is analyzed, it is seen that in many countries, financing has become increasingly mixed, with taxes, social insurance premiums, household out-of-pocket payments and private health insurance.

Liu and Zhang [4] analyzed provincial government health input dynamic efficiency by Malmquist index method and examined whether there exists convergence of the efficiency of the eastern middle and west provinces. Hadad et al. [3] analyzed the technical efficiency of health production across the OECD countries, using the data envelopment analysis (DEA) method. In their study, Data envelopment analysis (DEA) was utilized to calculate OECD countries' healthcare system efficiency. Life expectancy and infant survival rate were considered as outputs in both models. Healthcare

systems' rankings according to the super-efficiency and the cross-efficiency ranking methods were used to analyze determinants associated with efficiency. Gearhart [5] re-examined analyses of cross-country healthcare efficiency using modern, non-parametric estimators and Malmquist indices to determine productivity changes over the panel. Cheng et al. [7] analyzed the efficiency and productivity changes in hospitals before and after the reform process. First, they performed bootstrapping data envelopment analysis (DEA) to estimate the technical efficiency (TE), pure technical efficiency (PTE) and scale efficiency (SE) of the sample hospitals during the period. Second, they used the bootstrapping Malmquist productivity index to calculate the productivity changes over time. Kim et al. [8] evaluated productivity changes in the healthcare systems of 30 OECD countries over the 2002–2012 periods. They used the bootstrapped Malmquist approach to estimate bias-corrected indices of healthcare performance in productivity, efficiency and technology by modifying the original distance functions. Kohl et al. [6] reviewed 262 papers of DEA applications in healthcare with special focus on hospitals and therefore closes a gap of over ten years that were not covered by existing review articles. The aim of this study is to examine to the case of Malmquist indices constructed from nonparametric distance function estimates using data of OECD countries during the period of 2010 through 2014.

## II. MATERIALS AND METHOD

DEA, as developed by Charnes et al. [9] and extended by Banker et al. [10] is a linear programming technique based on approach for measuring the relative efficiencies. The efficiency of a Decision-Making Units (DMU) is expressed in terms of a set of measures which are classified as DEA inputs and outputs [11].

Suppose we have a set of  $n$  DMUs in the model, and each DMU has  $m$  inputs and  $s$  outputs. The CCR model can be mathematically expressed as (1).

$$\begin{aligned} & \text{Max } \frac{\sum_{r=1}^s u_{rk} Y_{rj}}{\sum_{i=1}^m v_{ik} X_{ij}} \\ & \text{Subject to:} \\ & \frac{\sum_{r=1}^s u_{rk} Y_{rj}}{\sum_{i=1}^m v_{ik} X_{ij}} \leq 1 \quad \text{for all } j \end{aligned} \quad (1)$$

$$u_{rk} \geq 0, v_{ik} \geq 0 \quad \text{for all } r, k$$

$Y_{rj}$  = the vector of output  $r$  produced by unit  $j$ ,  
 $X_{ij}$  = the vector of input  $i$  used by unit  $j$ ,  
 $u_{rk}$  = the weight given to output  $r$  by the base unit  $k$   
 $v_{ik}$  = the weight given to input  $i$  by the base unit  $k$   
 $(r = 1, \dots, s), (i = 1, \dots, m)$

The DEA relative efficiency measure for a target decision making unit  $k$  can be determined by solving the CCR (Charnes, Cooper and Rhodes) or BCC (Banker, Charnes, Cooper) models. CCR model calculates the efficiency ratio for the DMUs based on their inputs and outputs and it is under constant returns to scale (CRS) technology which are inputs and outputs linked in a strictly proportional manner. The others BCC model is under variable returns to scale (VRS) technology and it estimates the pure technical efficiency of a DMU at a given scale of operation. The only difference between the CCR and BCC models is the convexity condition of the BCC model, which means that the frontiers of the BCC model have piecewise linear and concave characteristics, which lead to variable returns to scale [12].

DEA assigns an efficiency score, one to efficient units and less than one to inefficient units. Then, it evaluates the technical efficiency of DMUs but doesn't allow for a ranking of the efficient units themselves. When the data of evaluated DMUs is the panel data containing more than one observed values of time points, the changes in productivity and the respective roles of technical efficiency and technical progress on the productivity changes can be analyzed, which is the commonly-used Malmquist total factor productivity index analysis. The concept of Malmquist productivity index is originally derived from [16], so this type of index is named Malmquist Index. [17] first calculated the Malmquist Index by using the DEA method, and decomposed the Malmquist Index into two aspects of change: one is the change of technical efficiency of the evaluated DMU in two periods; the other is the change of production technology, reflecting the changes of the production frontier in the DEA analysis. A weakness with the standard DEA model is that it doesn't incorporate any random noise. The method only gives point estimates of efficiency and productivity that do not offer any information of the uncertainty in the country-specific estimates. The bootstrap can be implemented in various ways of efficiency and Malmquist indices [13].

This paper analyzes the bootstrap method for calculation of confidence intervals for OECD countries DEA-based Malmquist productivity indices under the assumption of the

constant returns to scale to measure efficiencies of healthcare. Our analysis proceeds in three steps. The first step involves the application of DEA to obtain year-by-year efficiency scores. The second step involves the calculation of the Malmquist productivity index (MPI) and its components to obtain information on performance changes over time. The third step applies a bootstrapping technique to as certain confidence intervals for the obtained efficiency and productivity scores. With this aim, we consider two outputs: infant survival rate ( $y_1$ ) and life expectancy ( $y_2$ ), and five inputs: number of doctors ( $x_1$ ), number of nurses ( $x_2$ ), health expenditures in GDP ( $x_3$ ), number of beds ( $x_4$ ) and number of hospitals ( $x_5$ ). Our data consist of 21 OECD countries healthcare dataset over the period 1987–1997. Table 1 illustrates that the main descriptive statistics for variables used in the study.

**Table 1.** Basic statistics of inputs and outputs

Variables		Mean	Min	Max	Std.Dev.
Number of Doctors	2010	3,00	1,70	4,80	0,76
	2011	3,05	1,71	4,84	0,73
	2012	3,08	1,73	4,90	0,73
	2013	3,15	1,75	4,99	0,74
	2014	3,19	1,75	5,05	0,75
Number of Nurses	2010	8,14	1,60	16,03	3,94
	2011	8,44	1,70	16,60	3,91
	2012	8,55	1,80	16,97	4,00
	2013	8,59	1,80	17,36	4,15
	2014	8,72	1,90	17,56	4,17
Health Expenditures in GDP	2010	8,76	5,05	16,40	2,49
	2011	8,65	4,69	16,36	2,55
	2012	8,74	4,48	16,36	2,57
	2013	8,77	4,40	16,32	2,58
Number of Beds	2010	4,81	1,59	8,74	2,02
	2011	4,89	1,59	9,53	2,10
	2012	4,89	1,57	10,25	2,17
	2013	4,84	1,62	10,92	2,25
	2014	4,81	1,62	11,59	2,33
Number of Hospitals	2010	30,21	11,28	8,74	12,71
	2011	29,58	11,20	9,53	13,08
	2012	29,23	10,87	10,25	13,50
	2013	28,77	10,67	10,92	13,86
	2014	28,49	10,35	11,59	14,35
Infant survival rate	2010	262,69	69,92	453,55	100,21
	2011	297,31	71,99	1110,11	202,67
	2012	307,22	74,19	908,09	178,05
	2013	293,35	75,92	554,56	123,45
	2014	300,15	79,00	554,56	113,51
Life expectancy	2010	79,36	73,00	82,60	3,04
	2011	79,70	73,70	82,80	2,93
	2012	79,81	73,90	83,00	2,92
	2013	80,20	74,10	83,20	2,76
	2014	80,52	74,30	83,30	2,76

## III. RESULTS

First, I calculated the input-oriented MPI of productivity changes and compared results across units and time periods. All computations were analysed by Max – DEA Ultra program and the results are listed in Table 2 which shows the technic efficiency, technological efficiency and Malmquist Productivity Index of the OECD countries.

**Table 2.** Technic, technological and Malmquist efficiency scores

DMU	Arithmetic Technic Efficiency	Arithmetic Technologic Efficiency	Arithmetic Malmquist Productivity Index
Austria	0,9952	1,0274	1,0166
Belgium	1,0210	0,9879	1,0086
Canada	1,0000	1,0046	1,0046
Estonia	1,0011	1,0534	1,0573
Finland	0,9824	1,0425	0,9950
France	1,0006	0,9996	0,9975
Germany	0,9748	1,0259	0,9909
Hungary	0,9954	1,0003	0,9956
Iceland	1,0000	1,0921	1,0921
Israel	1,0000	1,0141	1,0141
Italy	0,9993	1,0131	1,0096
Korea	1,0000	0,9819	0,9819
Latvia	1,0223	1,0341	1,0542
Luxembourg	1,0044	1,0324	1,0344
Mexico	1,0000	0,9991	0,9991
New Zealand	1,0058	0,9942	0,9987
Slovenia	1,0000	1,0130	1,0130
Spain	1,0000	1,0216	1,0216
Switzerland	0,9895	1,0220	1,0057
Turkey	1,0000	1,0072	1,0072
United States	1,0088	1,0042	1,0128

Once efficiency scores and productivity indices are obtained, I must still obtain the appropriate confidence intervals for the derived scores and determine whether the obtained scores and indices are significant. Table 3 presents summary results for the first-stage input saving DEA efficiency scores, bootstrapped scores, and the frequency distribution for returns to scale technologies.

**Table 3.** Summary results of original and bootstrapped efficiency scores

	2010	2011	2012	2013	2014	Mean
Mean	0,9072	0,8693	0,8744	0,8936	0,9022	0,8893
Min	0,6463	0,5498	0,5608	0,5884	0,5724	0,5835
Std.Dev.	0,1268	0,1469	0,1438	0,1368	0,1383	0,1385
Bootstrap Sample						
Mean	0,8193	0,7681	0,7735	0,7942	0,8130	0,7936
Std.Dev.	0,0781	0,0828	0,0843	0,0837	0,0785	0,0815
Confidence Interval %5						
Lower bound	0,7419	0,6808	0,6858	0,7075	0,7349	0,7102
Upper bound	1,0034	0,9644	0,9727	0,9864	1,0083	0,9871
Distribution						
<70	3	4	4	3	4	
71-80	1	1	2	2	2	
81-90	4	5	4	4	2	
91-100	13	11	11	12	13	

The upper panel of Table 3 presents the original non-bootstrapped efficiency scores. The mean efficiency score across sample years is 0.8893. This implies that the mean potential for input savings among OECD is about 12%. For individual years, we observe fluctuations in average scores. Looking now at the distribution of efficiency scores across individual countries in the lowest panel of the table, we observe that the number of countries with efficiency scores in

the 91–100 interval follows the same trend as above. The number of countries in this range is respectably high, exceeding 11 and 13 in most available years. The bootstrap results for these average levels are shown in the middle panel of the table. We resampled at  $B = 500$ , yielding 500 pseudo-samples. From these results, we conclude that the means are within relatively large confidence intervals. This indicates that there is wide variation in efficiency scores across years, as can be observed in the standard deviations of the original estimates in the upper panel of the table. One important aspect of deriving confidence intervals is the certainty with which we can say that mean efficiency scores between years actually differ. There is overlap between almost all mean efficiency scores' confidence intervals, implying that we cannot simply assert that they are different across years, even the original scores appear different. Consequently, as stated by [15], one should be careful when making performance comparisons based on original efficiency scores [14].

#### IV. DISCUSSION

In this study, the effectiveness of health care services of OECD countries is discussed. When the results of the Malmquist productivity index used due to panel data are considered, the highest improvement belongs to Iceland, but in general, the productivity of the countries has been close to each other by years. Therefore, when we look at the Bootstrap DEA results, which are used for the non-decision making process due to the nature of the data envelopment analysis, it is seen that the lower and upper limits of the activities are in a wide range. In subsequent studies, regression analyzes can be performed by considering the environmental factors affecting the efficiency values.

#### V. CONCLUSION

This full paper contributes to the literature on efficiency and productivity measurement in the healthcare of OECD countries by employing Malmquist indices to measure productivity, DEA to measure efficiency, and bootstrapping to ascertain confidence intervals for the estimators. In this view, this paper provides an alternative and complementary approach to performance assessment, as well as to the determinants of that performance. The analysis therefore contributes to knowledge about the efficiency and productivity of OECD countries. Finally, non-parametric estimations, in combination with bootstrapping techniques, can help avoid the ambiguities present in conventional DEA analyses.

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