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A Weighted Mean-Squared Error Optimization Model with both Controllable and Noise Input Variables for a Cuboidal Design Region

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Abstract – A central composite design is a good choice for a spherical design region while providing high-quality predictions over the entire spherical design region. However, this design requires design variable settings outside the range of the design variables in the factorial part. On the other hand, a face-centered design provides high-quality prediction over the entire cuboidal design region and does not require using design points outside the factorial ranges. Therefore, a face-centered design is preferred over other designs. In the literature, controllable input variables have been addressed. However, both controllable and noise input variables have been paid little attention. The aim is to build regression models for both the process mean and variance. The next task is to obtain an optimal operating condition for both controllable and noise input variables. A weighted mean-squared error optimization model is proposed. Comparison studies are conducted while considering different weights for each component of the objective function. Finally, the proposed methodology is an effective technique to obtain optimal settings for a cuboidal design region.

Keywords – *Quality Engineering, Weighted Mean-Squared Error Model, Controllable Input Variables, Noise Input Variables, Face-Centered Design, Optimization*

I. INTRODUCTION

Optimization is an important phase for the design of the experiment. For this particular purpose, Vining and Myers [1] offered the dual response model-based robust parameter design (RPD) by using the idea from Myers and Carter [2]. The model has minimized the estimated standard deviation when the estimated mean value is brought to the target value as nominal the best. The model is given below:

$$\text{minimize } \hat{\sigma}(\mathbf{x})$$
$$\text{subject to } \hat{\mu}(\mathbf{x}) = \mu_\tau$$
$$LB \le x_i \le UB \ (i = 1, 2, ..., n); \forall x_i \in R$$

where $\hat{\sigma}(x)$ is the estimated standard deviation, and $\hat{\mu}(x)$ is the estimated mean value. μ_τ is the desired target value. *LB* and *UB* are lower, and upper bound, respectively. In addition, x_i is the i^{th} independent variable. This approach was improved by Del Castillo and Montgomery [3], and they applied to get a solution by using a generalized reduced gradient (GRG) algorithm with inequality constraints. After that, DRM was developed by considering the process bias by Lin and Tu [4]. They offered to formulate RPD with the mean square error (MSE) approach. Lin and Tu [4] basically minimized MSE, and this model also allows the process bias. The model is expressed as:

$$\text{minimize } [\hat{\mu}(\mathbf{x}) - \mu_\tau]^2 + [\hat{\sigma}(\mathbf{x})]^2$$
$$\text{subject to } LB \le x_i \le UB \ (i = 1, 2, ..., n); \forall x_i \in R$$

Copeland and Nelson [5] improved the MSE model with the desired distance approach into RPD. In addition, they applied to the Nelder-Mead simplex algorithm to get the optimal solution. The model is shown below:

$$\text{minimize } \hat{\sigma}(\mathbf{x})$$
$$\text{subject to } [\hat{\mu}(\mathbf{x}) - \mu_\tau]^2 \le \Delta^2$$
$$LB \le x_i \le UB \ (i = 1, 2, ..., n); \forall x_i \in R$$

where Δ is the desired distance.

Koksoy and Doganaksoy [6] studied the weighted MSE approach for controllable input variables. In addition, Koksoy and Doganaksoy [6] applied to Pareto optimal solutions to produce more variant solutions. The weighted MSE model is stated for controllable input variables as follows:

$$\text{minimize } w_1[\hat{\mu}(x) - \mu_\tau]^2 + w_2[\hat{\sigma}(x)]^2$$
$$\text{subject to } w_1 + w_2 = 1$$
$$w_1 \ge 0, w_2 \ge 0$$

The recent studies in the RPD framework were conducted by Ouyang et al. [7], Ozdemir and Cho [8, 9, 10], Tsai and Liukkonen [11], Hot et al. [12], Lu et al. [13] and Chatterjee et al. [14].

This paper is three-fold. One, a face-centered design (FCD) is used for both controllable and noise input variables. Two, a regression model is built for both controllable and noise input variables in order to estimate the process mean and variance responses. Three, a weighted mean-squared error (WMSE) model is presented to obtain optimal settings for both controllable and noise input variables.

This paper is organized as follows. First of all, the proposed methodology development is presented in Section II. Then, a numerical example is conducted in Section III. Finally, concluding remarks are drawn in Section IV.

II. PROPOSED METHODOLOGY DEVELOPMENT

The proposed methodology development consists of the three main phases: (1) the design phase, (2) the modeling phase, (3) the optimization phase.

A. The Design Phase

There are some situations where the design region is cuboidal rather than spherical. In these cases, an appropriate central composite design (CCD) is the face-centered design (FCD) with the axial point=1. The FCD is not rotatable. The FCDs do not require as many center points as the spherical central composite design. The FCD consists of factorial, axial, and center points. In practice, two center points are sufficient for the FCD. Note that axial points are -1 and +1.

B. The Modeling Phase

For controllable input variables, the general form of the second-order regression function is given as follows:

$$f_x[\mathbf{x},\boldsymbol{\beta}_x]=\sum_{i=1}^{l}\beta_i x_i+\sum_{i<j=2}^{l}\sum_{i=1}^{l}\beta_{ij}x_i x_j+\sum_{i=1}^{l}\beta_{ii}x_i^2$$

where x_i is the i^{th} controllable input variable and β is the regression coefficient.

For noise input variables, the regression function is

$$f_u[\mathbf{x},\ \mathbf{u},\ \boldsymbol{\alpha}_u,\ \boldsymbol{\delta}_{xu}]=\sum_{j=1}^{m}\alpha_j u_j+\sum_{j=1}^{m}\sum_{i=1}^{l}\delta_{ij}x_i u_j$$

where u_j is the j^{th} noise input variable, α and δ represent the regression coefficients.

Then, the second-order regression model is denoted for both controllable and noise input variables as follows:

$$y=f_x[\mathbf{x},\boldsymbol{\beta}_x]+f_u[\mathbf{x},\ \mathbf{u},\ \boldsymbol{\alpha}_u,\ \boldsymbol{\delta}_{xu}]$$
$$=\sum_{i=1}^{l}\beta_i x_i+\sum_{i<j=2}^{l}\sum_{i=1}^{l}\beta_{ij}x_i x_j+\sum_{i=1}^{l}\beta_{ii}x_i^2+\sum_{j=1}^{m}\alpha_j u_j+\sum_{j=1}^{m}\sum_{i=1}^{l}\delta_{ij}x_i u_j$$

In addition, the fitted functions of mean, standard deviation and variance incorporated the model matrix, ***X***, from the *I*-optimality criterion for both controllable and noise input variables are found as follows:

$$\hat{\mu}[\mathbf{x},\mathbf{u}]=\mathbf{X}[\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'\bar{\mathbf{y}}\ \text{where}\ \bar{\mathbf{y}}=[\bar{y}_1,\ \bar{y}_2,\ ...,\bar{y}_n]$$

$$\hat{\sigma}[\mathbf{x},\mathbf{u}]=\mathbf{X}[\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{s}\ \text{where}\ \mathbf{s}=[s_1,\ s_2,\ ...,s_n]$$

$$\hat{\sigma}^2[\mathbf{x},\mathbf{u}]=\mathbf{X}[\mathbf{X}'\mathbf{X}]^{-1}\mathbf{X}'\mathbf{s}^2\ \text{where}\ \mathbf{s}^2=[s_1^2,\ s_2^2,\ ...,s_n^2]$$

where $\bar{\mathbf{y}}$ is the vector for the mean of each run, $\mathbf{s}$ is the vector for the standard deviation of each run, $\mathbf{s}^2$ is the vector for the variance of each run, and

$$\mathbf{X}=\begin{pmatrix}1 & x_{11} & \cdots & x_{n1} & x_{11}^2 & \cdots & x_{n1}^2 & x_{11}x_{21} & \cdots & x_{n-11}x_{n1} & u_{11} & \ldots & u_{n1} & x_{11}u_{11} & \ldots & x_{n1}u_{n1}\\ 1 & x_{12} & \cdots & x_{n2} & x_{12}^2 & \cdots & x_{n2}^2 & x_{12}x_{22} & \cdots & x_{n-12}x_{n2} & u_{12} & \ldots & u_{n2} & x_{12}u_{12} & \ldots & x_{n2}u_{n2}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 1 & x_{1l} & \cdots & x_{nl} & x_{1l}^2 & \cdots & x_{nl}^2 & x_{1l}x_{2l} & \cdots & x_{n-1l}x_{nl} & u_{1m} & \ldots & u_{nm} & x_{1l}u_{1m} & \ldots & x_{nl}u_{nm}\end{pmatrix}.$$

C. The Optimization Phase

The aim of the proposed optimization model is to minimize a weighted mean-squared error function for both controllable and noise input variables.

$$\text{Min Z}=w_1\left[\hat{\mu}[\mathbf{x},\mathbf{u}]-\mu_\tau\right]^2+w_2\hat{\sigma}^2[\mathbf{x},\mathbf{u}]$$

The constraints are

$$w_1+w_2=1$$
$$w_1\geq 0, w_2\geq 0$$
$$-1\leq x_i\leq +1\ \left(i=1,2,...,n\ \text{and}\ \forall x_i\in R\right)$$
$$-1\leq u_j\leq +1\ \left(j=1,2,...,m\ \text{and}\ \forall u_j\in R\right)$$

where w_1 and w_2 are the weights for each term in the objective function.

The model is able to minimize the process variance and the process bias at the same time. The model success based on the weights for each term in the objective function.

III. NUMERICAL EXAMPLE

A numerical example is presented in order to show the effectiveness of the proposed methodology. The numerical example is modified from Myers et al. [15]. The two input variables and the noise input variable, and their levels are shown in Table 1.

Table 1. Input variables and their levels

Level	Agitation (x_1)	Rate (x_2)	Temperature (u_1)
High (+1)	10.0	25	200
Center (0)	7.50	20	175
Low (-1)	5.0	15	150

The FCD is conducted and the collected data are given in Table 2.

Table 2. The collected data for the experiment

Run	x_1	x_2	u_1	y_1	y_2	y_3	$\bar{y}$	s	s^2
1	-1	-1	-1	50	56	53	53	3.0	9.0
2	-1	-1	+1	56	58	60	58	2.0	4.0
3	-1	+1	-1	58	57	64	59	3.8	14.3
4	-1	+1	+1	57	55	56	56	1.0	1.0
5	+1	-1	-1	62	66	64	64	2.0	4.0
6	+1	-1	+1	43	48	44	45	2.6	7.0
7	+1	+1	-1	38	30	37	35	4.4	19.0
8	+1	+1	+1	57	62	61	60	2.6	7.0
9	-1	0	0	64	55	58	59	4.6	21.0
10	+1	0	0	60	70	62	64	5.3	28.0
11	0	-1	0	50	55	54	53	2.6	7.0
12	0	+1	0	61	63	71	65	5.3	28.0
13	0	0	0	63	67	65	65	2.0	4.0
14	0	0	0	57	61	59	59	2.0	4.0
15	0	0	0	63	61	62	62	1.0	1.0
16	0	0	0	60	61	62	61	1.0	1.0

Notice that there are no axial points for the noise input variable in Table 2. Therefore, the FCD is modified based on the noise input variable. The estimated mean and variance responses are denoted as follows:

$$\hat{\mu}[\mathbf{x},\mathbf{u}] = 62.7 - 1.7x_1 + 0.2x_2 - 2.3x_1x_2 - 2.9x_1^2 - 5.5x_2^2 + 1.0u_1 + 0.5x_1u_1 + 4.5x_2u_1$$

$$\hat{\sigma}^2[\mathbf{x},\mathbf{u}] = 8.2 + 1.6x_1 + 3.8x_2 + 1.6x_1x_2 + 4.9x_1^2 - 2.1x_2^2 - 3.4u_1 + 1.2x_1u_1 - 2.9x_2u_1$$

The proposed optimization model is given as follows:

$$\text{Min Z}=w_1\left[\begin{matrix}62.7 - 1.7x_1 + 0.2x_2 - 2.3x_1x_2 - 2.9x_1^2 - 5.5x_2^2 \\ +1.0u_1 + 0.5x_1u_1 + 4.5x_2u_1 - \mu_\tau\end{matrix}\right]^2 + w_2\left[\begin{matrix}8.2 + 1.6x_1 + 3.8x_2 + 1.6x_1x_2 + 4.9x_1^2 - 2.1x_2^2 \\ -3.4u_1 + 1.2x_1u_1 - 2.9x_2u_1\end{matrix}\right]$$

subject to

$$w_1 + w_2 = 1$$

$$w_1 \geq 0, w_2 \geq 0$$

$$-1 \leq x_1, x_2, u_1 \leq +1 \text{ and } x_1, x_2, u_1 \in R$$

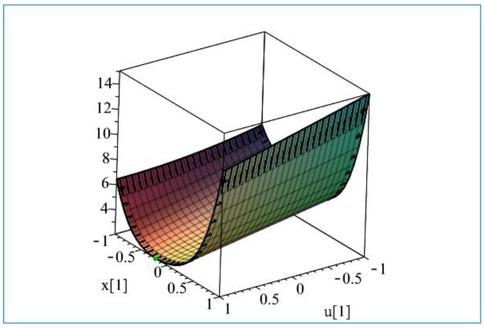
In Table 3, the results of the proposed WMSE model are shown with different weights for each term in the objective function.

Table 3. The results of the proposed WMSE model

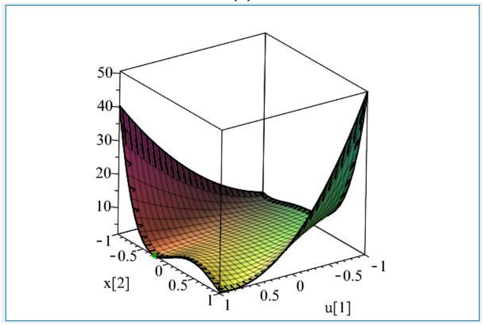
Case	Weights	x_1	x_2	u_1	Min Z
1	w_1=0.0 w_2=1.0	-0.449	1.000	1.000	2.612
2	w_1=0.1 w_2=0.9	-0.201	-0.393	1.000	3.606
3	w_1=0.2 w_2=0.8	-0.215	-0.324	1.000	3.257
4	w_1=0.3, w_2=0.7	-0.220	-0.302	1.000	2.864
5	w_1=0.4 w_2=0.6	-0.222	-0.291	1.000	2.460
6	w_1=0.5 w_2=0.5	-0.223	-0.284	1.000	2.053
7	w_1=0.6 w_2=0.4	-0.224	-0.280	1.000	1.644
8	w_1=0.7 w_2=0.3	-0.225	-0.277	1.000	1.234
9	w_1=0.8 w_2=0.2	-0.225	-0.275	1.000	0.823
10	w_1=0.9 w_2=0.1	-0.226	-0.273	1.000	0.412
11	w_1=1.0 w_2=0.0	-0.010	-0.285	0.718	0.000

The results show that the noise input variable is 1.000 for each case except the eleventh case. Between the fourth and the tenth cases, the results are very close to the controllable input variables. The sixth case is used when the weights for the terms in the objective function are equal. If the process bias, $\hat{\mu}[\mathbf{x},\mathbf{u}] - \mu_\tau$, is the priority, the cases (7-11) are appropriate for the experiment. If the process variance is the priority, the cases (1-5) are suitable for the experiment.

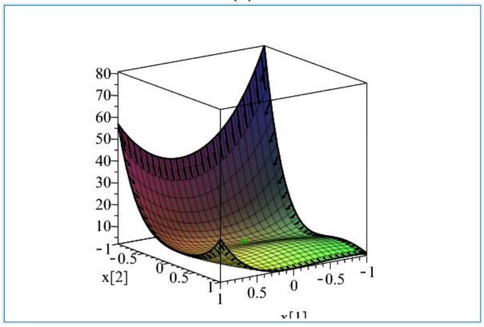
Fig. 1 shows the response plots for the sixth case. Note that the green point for each plot represents the optimal design point for the cuboidal design region.



(a)



(b)



(c)

Fig. 1 Response plots of the sixth case for (a) x_1 and u_1; (b) x_2 and u_1; (c) x_1 and x_2

IV. Conclusion

This paper presents a proposed methodology in order to obtain optimal settings for a cuboidal design region. For this purpose, an FCD is modified for noise input variables. The axial points are not used for noise input variables. Then, a second-order regression model is built for both controllable and noise input variables. Thus, the estimated process mean and variance are found. The final phase is to propose a WMSE model. A numerical example is used. The eleven different cases are analyzed. The results show that the proposed WMSE is able to achieve to obtain optimal settings for each case.

For further study, a multi-objective WMSE could be proposed for two or more responses. Next, the maximum allowed process bias could be incorporated into the proposed WMSE optimization model.

References

[1] G. G. Vining and R. H. Myers, "Combining Taguchi and response surface philosophies: a dual response approach," *J. Qual. Technol.*, vol. 22(1), pp. 38-45, 1990.

[2] R. H. Myers and W. H. Carter, "Response surface techniques for dual response systems," *Technometrics*, vol. 15(2), pp. 301-317, 1973.

[3] E. Del Castillo and D. C. Montgomery, "A nonlinear programming solution to the dual response problem," *J. Qual. Technol.*, vol. 25(3), pp. 199-204, 1993.

[4] D. K. Lin and W. Tu, "Dual response surface optimization," *J. Qual. Technol.*, vol. 27(1), pp. 34-39, 1995.

[5] K. A. Copeland and P. R. Nelson, "Dual response optimization via direct function minimization," *J. Qual. Technol.*, vol. 28(3), pp. 331-336, 1996.

[6] O. Köksoy and N. Doganaksoy, "Joint optimization of mean and standard deviation using response surface methods," *J. Qual. Technol.*, vol. 35(3), pp. 239-252, 2003.

[7] L. Ouyang, Y. Ma, J. H. Byun, J. Wang, and Y. Tu, "An interval approach to robust design with parameter uncertainty," *Int. J. Prod. Res.*, vol. 54(11), pp. 3201-3215, 2016.

[8] A. Ozdemir and B. R. Cho, "A Nonlinear integer programming approach to solving the robust parameter design optimization problem," *Qual. Reliab. Eng. Int.*, vol. 32(8), pp. 2859-2870, 2016.

[9] A. Ozdemir and B. R. Cho, "Response surface-based robust parameter design optimization with both qualitative and quantitative variables." *Eng. Optimiz.*, vol. 49(10), pp. 1796-1812, 2017.

[10] A. Ozdemir and B. R. Cho, "Response surface optimization for a nonlinearly constrained irregular experimental design space," *Eng. Optimiz.*, vol. 51(12), pp. 2030-2048, 2019.

[11] T. N. Tsai and M. Liukkonen, "Robust parameter design for the micro-BGA stencil printing process using a fuzzy logic-based Taguchi method," *Appl. Soft Comput.*, vol. 48, pp. 124-136, 2016.

[12] A. Hot, T. Weisser, and S. Cogan, "An info-gap application to robust design of a prestressed space structure under epistemic uncertainties," *Mech. Syst. Signal Pr.*, vol. 91, pp. 1-9, 2017.

[13] Y. Lu, S. Wang, C. Yan, and Z. Huang, "Robust optimal design of renewable energy system in nearly/net zero energy buildings under uncertainties," *Appl. Energ.*, vol. 187, pp. 62-71, 2017.

[14] K. Chatterjee, K. Drosou, S. D. Georgiou, and C. Koukouvinos, "Response modelling approach to robust parameter design methodology using supersaturated designs," *J. Qual. Technol.*, vol. 50(1), pp. 66-75, 2018.

[15] R. H. Myers, D. C. Montgomery, and C. M. Anderson-Cook, *Response Surface Methodology: Process and Product Optimization Using Designed Experiments (Wiley Series in Probability and Statistics)*, Hoboken, NJ: Wiley, 2016.