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SOLVING A CLASS OF TRANSMISSION PROBLEMS BY THE DIFFERENTIAL TRANSFORM METHOD

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Abstract : There are various numerical methods for solving many problems in mathematical physics. The differential transform method (DTM) is one of the numerical methods for solving ordinary and partial differential equations. The concept of the differential transform was first proposed by Zhou. Zhou's aim was to solve both linear and non-linear initial value problems in electric circuit analysis. In this work, we study the performance of the DTM applied to the solution of boundary value problems for Sturm-Liouville equation with additional transmission conditions at one interior singular point. Note that, the present method can be applied to many initial value problems and boundary-value problems with additional transmission conditions and does not require linearization, perturbation or discretization.

Keywords : Differential transform method, transmission conditions, approximation solution

1 Introduction

The differential transform method (DTM) is a aproximation method for solving various type linear and nonlinear differential equations. This method was first proposed by Zhou [1] for solving both linear and nonlinear initial value problems in electric circuit analysis. Saravanan and Magesh have carried out the comparative study between Adomian decopmosition method (ADM) and DTM by handling the Newell-Whitehead-Segel equation, in [2].

In [3], the DTM is applied for solving integral equations Using DTM to solve the Lane-Emden type equations is introduced in [4]. Kanth and Aruna applyied the DTM for solving singular boundary value problems in [5].

The goal of the [6] paper is to provide a new technique based on DTM to equations involving non-linearities. The DTM which is applied to solve eigenvalue problems is proposed in [7] to find the eigenvalues and the normalized eigenfunctions for the second- and the fourth order differential equations. In [8], a numerical method based on DTM is used to solve the Bratu problem.

In this study, we found the approximate solution of initial value problems with additional transmission conditions based on DTM.

2 Solution of one boundary value transmission problem by DTM

We shall consider the differential equation,

$$y''(x) + y(x) = 0, \qquad x \in [0,1) \cup (1,2] \tag{2.1}$$

subject to initial-transmission conditions,

$$y(0) = 1, \qquad y'(0) = 0 \tag{2.2}$$

$$y(1-0) = y(1+0), \quad y'(1-0) = \gamma_2 y'(1+0) \tag{2.3}$$

First, let's get the solution for the problem in the left interval $x \in [0,1)$. If differential transform method is applied to the differential equation,

$$Y^-(k+2) = \frac{-Y^-(k)}{(k+2)(k+1)} \tag{2.4}$$

is obtained. Then, using the representation $y^-(x) = \sum_{k=0}^{n}(x-x_0)^k Y^-(k)|_{x=x_0}$, $y(1) = 0$, the following transformed initial condition at $x_0 = 0$ can be obtained as $Y^-(0) = 1$, $Y^-(1) = 0$ Taking in view

$$y'^-(x) = Y^-(1) + 2xY^-(2) + ... + nx^{n-1}Y^-(n) \tag{2.5}$$

and $y'(0) = 0$ and then substituting we have $Y^-(2) = \frac{-1}{2}$, $Y^-(3) = 0$, $Y^-(4) = \frac{1}{4!}$, $Y^-(5) = 0$, $Y^-(6) = \frac{-1}{6!}$, ...

Let's choose $n = 6$. Thus, we get the following equations

$$y^-(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}, \; y'^-(x) = -x + \frac{x^3}{3!} - \frac{x^5}{5!}$$

Secondly, let's get the solution for the problem in the $x \in (1,2]$. If differential transform method is applied to the differential equation,

$$Y^+(k+2) = \frac{-Y^+(k)}{(k+2)(k+1)} \tag{2.6}$$

is obtained for $x_0 = 2$. Using

$$y^+(x) = Y^+(0) + (x-2)Y^+(1) + ... + (x-2)^n Y^+(n) \tag{2.7}$$

and following the similar recursive procedure, we find

$$\begin{aligned} y^+(x) &= M + (x-2)N + \frac{-M}{2!}(x-2)^2 + \frac{-N}{3!}(x-2)^3 \\ &+ \frac{M}{4!}(x-2)^4 + \frac{N}{5!}(x-2)^5 + \frac{-M}{6!}(x-2)^6 \end{aligned} \tag{2.8}$$

$$\begin{aligned} y'^+(x) &= N - M(x-2) + \frac{N}{2!}(x-2)^2 \\ &+ \frac{M}{3!}(x-2)^3 + \frac{N}{4!}(x-2)^4 + \frac{-M}{5!}(x-2)^5 \end{aligned} \tag{2.9}$$

Hence we have

$$1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} = M - N - \frac{M}{2!} + \frac{N}{3!} + \frac{M}{4!} - \frac{N}{5!} - \frac{M}{6!} \tag{2.10}$$

$$-1 + \frac{1}{3!} - \frac{1}{5!} = \gamma_2 \left(N + M + \frac{N}{2!} - \frac{M}{3!} + \frac{N}{4!} + \frac{M}{5!} \right) \tag{2.11}$$

By using these equalities we can find that

$$y(x) = \begin{cases} 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ..., & x \in [0,1) \\ 0.310593 + (-0.44254)(x-2) + \frac{-0.310593}{2!}(x-2)^2 + ... + \frac{-0.310593}{6!}(x-2)^6 + ..., & x \in (1,2] \end{cases}$$

3 Using differential transform method to solve nonlinear transmission problem

Let us consider the following nonlinear differential equation,

$$y''(x) + y^2(x) = 0, \qquad x \in [0,1) \cup (1,2]$$

subject to initial-transmission conditions,

$$y(0) = 1, \qquad y'(0) = 0$$

$$y(1-0) = y(1+0), \qquad y'(1-0) = 3y'(1+0)$$

First, let's get the solution for the problem in the $x \in [0,1)$. Then we have

$$Y^-(k+2) = \frac{-\sum_{r=0}^{k} Y(r)Y(k-r)}{(k+2)(k+1)} \tag{3.1}$$

for $x_0 = 0$, By using the series representation

$$y^-(x) = Y^-(0) + xY^-(1) + x^2Y^-(2) + ... + x^nY^-(n) \quad (3.2)$$

We can find that $Y^-(0) = 1$, $Y^-(1) = 0$, $Y^-(2) = \frac{-1}{2}$, $Y^-(3) = 0$, $Y^-(4) = \frac{1}{12}$, $Y^-(5) = 0$, $Y^-(6) = \frac{-1}{72}$, ...

In equation (3.2), let's choose $n = 6$. Then, we get the equations

$$y^-(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{12} - \frac{x^6}{72},\ y'^-(x) = -x + \frac{x^3}{3} - \frac{x^5}{12} \quad (3.3)$$

Secondly, let's get the solution for the problem in the $x \in (1, 2]$. If differential transform method is applied to the differential equation, we have

$$Y^+(k+2) = \frac{-\sum_{r=0}^{k} Y(r)Y(k-r)}{(k+2)(k+1)} \quad (3.4)$$

for $x_0 = 2$. Thus

$$y^+(x) = Y^+(0) + (x-2)Y^+(1) + ... + (x-2)^nY^+(n) \quad (3.5)$$

is written. Let's assume that $Y^+(0) = A$ and $Y^+(1) = B$.

Let us choose $n = 6$. Then we get

$$\begin{aligned} y^+(x) &= A + (x-2)B + \frac{-A^2}{2!}(x-2)^2 + \frac{-AB}{3}(x-2)^3 + \frac{A^3-B^2}{12}(x-2)^4 \\ &+ \frac{A^2B}{12}(x-2)^5 + \frac{-1}{30}(2A\frac{A^3-B^2}{12} + 2B(\frac{-A.B}{3}) + \frac{A^4}{4})(x-2)^6 \quad (3.6) \end{aligned}$$

$$\begin{aligned} y'^+(x) &= B - A^2(x-2) - A.B(x-2)^2 + \frac{A^3-B^2}{3}(x-2)^3 + \frac{5A^2B}{12}(x-2)^4 \\ &+ \frac{-1}{5}(2A\frac{A^3-B^2}{12} + 2B(\frac{-A.B}{3}) + \frac{A^4}{4})(x-2)^5 \quad (3.7) \end{aligned}$$

By using these equalities we have the following equations for finding A and B

$$\begin{aligned} 1 - \frac{1}{2!} + \frac{1}{12} - \frac{1}{72} &= A - B + \frac{-A^2}{2!} + \frac{A.B}{3} + \frac{A^3-B^2}{12} \\ &- \frac{A^2B}{12} + \frac{-1}{30}(2A\frac{A^3-B^2}{12} + 2B(\frac{-A.B}{3}) + \frac{A^4}{4}) \quad (3.8) \end{aligned}$$

$$\begin{aligned} -1 + \frac{1}{3} - \frac{1}{12} &= 3(B + A^2 - A.B - \frac{A^3-B^2}{3} + \frac{5A^2B}{12} \\ &+ \frac{1}{5}(2A\frac{A^3-B^2}{12} + 2B(\frac{-A.B}{3}) + \frac{A^4}{4})) \quad (3.9) \end{aligned}$$

Finally we have that

$$y(x) = \begin{cases} 1 - \frac{t^2}{2!} + \frac{t^4}{12} - \frac{t^6}{72} + ...,\ x \in [0, 1) \\ 0.185747 + (-0.44177)(x-2) + \frac{-(0.185747)^2}{2!}(x-2)^2 + ..., \quad x \in (1, 2] \end{cases}$$

Conclusion

In this study, we found the approximate solution of initial value problems with additional transmission conditions by using using DTM.

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