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# A Robust Formulation for U-shaped Assembly Line Balancing Problem Under Task Time Uncertainty by Considering Worker Skills

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***Abstract –*** The U-shaped assembly line balancing (UALB) problem has been extensively investigated in the existing academic literature. However, only a limited number of studies consider the uncertainty in the assembly lines. In this research, a robust formulation is developed for the addressed problem under task time uncertainty by focusing on the heterogeneity inherent of workers. The worker resources are not the same in terms of either skills or skill levels. Therefore, it is plausible to investigate the manufacturing system from this perspective to provide managerial insights regarding the performance metrics, such as the total cost, number of stations. Besides, the uncertainty of processes in a manufacturing system must be considered to improve system performance. Since robust optimization is a powerful technique to overcome uncertainty, the robust approach is employed in this study. First, the UALB problem is presented and followed by that the robust counterpart of this model is provided. The optimal results are obtained for different parameter levels after solving the model. The analysis of the results is made through real data provided by a water-meter assembly line.

***Keywords*** – *U-shaped assembly line balancing; Robust optimization; Worker skills, Worker proficiencies, Optimization models*

## I. INTRODUCTION

Today's intricate market environment necessitates focusing on assembly line problems from different perspectives. Assembly line balancing problem (ALBP) is one of the most important problems that give a path to increase the manufacturing system performance in terms of several factors, such as manufacturing lead-time, overall cost, and utilization of resources. While balancing an assembly line, the tasks are assigned to stations in such a way that the precedence and cycle time constraints are not violated. In this study, a U-shaped assembly line balancing problem with heterogeneity inherent of workers under uncertain environment is addressed. The U-shaped assembly lines pave the way for many advantages compared to traditional straight assembly lines, such as accepting new product types, improving workers' skill levels, allowing one-piece flow, less space requirement, and more flexibility to assign tasks to the stations [1]. The U-shaped assembly line balancing problem has been investigated in many studies [2], [3], [4], and [5]. When these studies are examined, it is revealed that most of them do not consider the uncertainty caused by the processes. However, some studies have employed several methods, such as fuzzy optimization, robust optimization, and stochastic optimization. Uncertainty issue was examined for both straight and U-shaped assembly lines by [6]. While uncertainty was handled, both the precedence and phantom networks were considered. These networks were introduced by [7]. The fuzzy assembly line balancing problem has been investigated by [8], [9], [10], and [11]. Besides, the U-shaped assembly line balancing problem was investigated by employing the stochastic optimization in [12], [13], [14], and [15].

The robust optimization is a powerful technique and superior to fuzzy and stochastic optimization in several ways. For instance, stochastic optimization is based on scenarios preventing to handle all cases. On the other hand, the robust optimization covers all cases simultaneously and does not require probability distribution information to find a solution for the worst-case. The robust optimization has been employed for the assembly line balancing problem in [16], [17], [18], and [19]. In this study, first, the nominal optimization model is formulated. Following by that, a robust counterpart of this model is constructed to overcome the uncertainty. The employed robust approach is first introduced by [18] and modified by [19]. There are important features for robust optimization: (i) tractability of the model, and (ii) conservatism. The first feature is important to get optimal solutions within reasonable computational time. The second feature is important to decrease the price of robustness. The robust method proposed by [20] is employed to ease the tractability of the optimization model and not to have over-conservative solutions. In this study, each aspect of robust optimization is investigated in the context of the addressed problem.

The rest of the paper is organized as follows. The optimization model is presented in Section 2. The computational results are given in Section 3. Conclusion and future research directions are provided in Section 4.

## II. OPTIMIZATION MODELS

In the following, the proposed robust optimization model for the U-shaped assembly line balancing problem with heterogeneity inherent of workers is formulated. The proposed model is extended by [3] to include uncertainty.

**Indices**

$i$: Index for tasks ($i$: 1,…,$I$)
$j$: Index for stations ($j$: 1,…,$J$)
$w$: Index for workers ($w$: 1,…,$W$)

**Parameters**

$n$ : number of tasks
$t_i$ : standard time for task $i$
$m_{max}$ : number of stations to be opened ($m_{max} \leq n$)
$C$ : cycle time
$W_j$ : tasks for station $j$
$||W_j||$ : number of tasks in $W_j$
$L(r,s)$ : a set for tasks that preceding task $s$
$p_{wi}$ : skill level of worker $w$ for task $i$
$Cost_j$ : cost for opening station $j$
$\Gamma$: number of uncertain task time

**Variables**

$A_{jw}$: if worker $w$ is assigned to station $j$, 1; otherwise, 0
$x_{ij}$ : if task $i$ is assigned to station $j$ for original network, 1; otherwise, 0
$y_{ij}$ : if task $i$ is assigned to station $j$ for phantom network, 1; otherwise, 0
$z_j$ : if station $j$ is opened, 1; otherwise 0

**Objective function**

$$\text{Min} \ \sum_{j=1}^{J} \cos t_j \times Z_j \tag{1}$$

$$\sum_{j=1}^{J} k_{ij} = 1 \qquad \forall i \tag{2}$$

$$x_{ij} + y_{ij} = k_{ij} \qquad \forall i, j \tag{3}$$

$$\sum_{i=1}^{I} t_i s_{ijw} p_{wi} + p \times \Gamma + \sum_{i=1}^{I} q_i \leq C \qquad \forall j, w \tag{4}$$

$$p + q_i \geq t_i yr_{ijw} p_{iw} \qquad \forall i, j, w \tag{5}$$

$$yr_{ijw} \geq s_{i,j,w} \qquad \forall i, j, w \tag{6}$$

$$A_{jw} k_{ij} = s_{ijw} \qquad \forall i, j, w \tag{7}$$

$$\sum_{j=1}^{J} (m_{\max} - j + 1)(x_{rj} - x_{sj}) \geq 0 \qquad \forall (r,s) \in L \tag{8}$$

$$\sum_{j=1}^{J} (m_{\max} - j + 1)(y_{rj} - y_{sj}) \geq 0 \qquad \forall (s,r) \in L \tag{9}$$

$$\sum_{i \in W_j} k_{ij} - \left\| W_j \right\| Z_j \leq 0 \qquad \forall j \tag{10}$$

$$\sum_{j=1}^{J} A_{jw} \leq 1 \qquad \forall w \tag{11}$$

$$\sum_{w=1}^{W} A_{jw} \leq 1 \qquad \forall j \tag{12}$$

$$Z_j - \sum_{w=1}^{W} A_{jw} = 0 \qquad \forall j \tag{13}$$

$$A_{jw} + k_{ij} \geq 2 s_{ijw} \qquad \forall i, j, w \tag{14}$$

$$A_{jw} + k_{ij} \leq 1 + s_{ijw} \qquad \forall i, j, w \tag{15}$$

$$x_{ij}, y_{ij}, Z_j, A_{jw}, s_{ijw}, yr_{ijw}, k_{ij} \in \{0,1\} \tag{16}$$

The objective function (1) aims to minimize total cost regarding the station utilization. Constraints (2) and (3) imply that each task must be assigned to the back or front of the U-shaped assembly line. Constraints (4), (5), and (6) are included in the model to obtain a robust counterpart. In this equation, the number of uncertain task time ($\Gamma$) is directly related to the probability of constraint violation (PoV). The parameter $\Gamma$ can take the values between 0 and $I$ that equals the number of tasks. As the parameter $\Gamma$ increases, PoV value decreases. The reason behind this fact is that when the considering number of uncertain task time increase, it means that the constraints are more protected against the violation. Constraints (8) and (9) ensure the precedence constraint for both precedence and phantom networks. Constraint (10) states that if a task assigned to a station, the station must be opened. Constraints (11), (12), and (13) indicate that a station is operated by one worker. In other words, multi-manned stations are not allowed. Constraints (14) and (15) protect the linearity of the proposed model. Constraint (16) defines 0-1 binary variables.

## III. COMPUTATIONAL RESULTS

In the following, the addressed problem is investigated from a scenario-based perspective. Three different scenarios are considered with respect to workers' heterogeneity. The scenarios are as follows:

Scenario-1: There is a worker pool in which 33% of total workers have high-skill level. 33% of total workers have moderate skill level, and 33% of total workers are low skill level.

Scenario-2: There is a worker pool in which 66% of total workers have high-skill level. 33% of total workers have low skill level.

Scenario-3: There is a worker pool in which 66% of total workers have low-skill level. 33% of total workers have high skill level.

Table 1. The results with respect to scenarios

| **UL-0.5** | | **WS-Scenario1** | **WS-Scenario2** | **WS-Scenario3** |
|---|---|---|---|---|
| **Γ** | **PoV** | **Cost** | **Cost** | **Cost** |
| 0 | 1,00 | 400 | 400 | 400 |
| 1 | 1,00 | 400 | 400 | 400 |
| 2 | 1,00 | 500 | 400 | 500 |
| 3 | 1,00 | 600 | 400 | 600 |
| 4 | 1,00 | 600 | 500 | 600 |
| 5 | 1,00 | 600 | 500 | 600 |
| 6 | 1,00 | 700 | 500 | 700 |
| 7 | 1,00 | 700 | 600 | 700 |
| 8 | 1,00 | 700 | 600 | 700 |
| 9 | 1,00 | 700 | 600 | 700 |
| **UL-0.25** | | **WS-Scenario1** | **WS-Scenario2** | **WS-Scenario3** |
| **Γ** | **PoV** | **Cost** | **Cost** | **Cost** |
| 0 | 1,00 | 400 | 400 | 400 |
| 1 | 1,00 | 400 | 400 | 400 |
| 2 | 1,00 | 400 | 400 | 500 |
| 3 | 1,00 | 400 | 400 | 500 |
| 4 | 1,00 | 500 | 400 | 600 |
| 5 | 1,00 | 600 | 500 | 600 |
| 6 | 1,00 | 600 | 500 | 600 |
| 7 | 1,00 | 600 | 500 | 700 |
| 8 | 1,00 | 700 | 600 | 700 |
| 9 | 1,00 | 700 | 600 | 700 |
| **UL-0.1** | | **WS-Scenario1** | **WS-Scenario2** | **WS-Scenario3** |
| **Γ** | **PoV** | **Cost** | **Cost** | **Cost** |

| | | | | |
|---|---|---|---|---|
| 0 | 1,00 | 400 | 400 | 400 |
| 1 | 1,00 | 400 | 400 | 400 |
| 2 | 1,00 | 400 | 400 | 500 |
| 3 | 1,00 | 400 | 400 | 500 |
| 4 | 1,00 | 500 | 400 | 500 |
| 5 | 1,00 | 600 | 400 | 600 |
| 6 | 1,00 | 600 | 400 | 600 |
| 7 | 1,00 | 600 | 400 | 700 |
| 8 | 1,00 | 700 | 400 | 700 |
| 9 | 1,00 | 700 | 500 | 700 |

UL: uncertainty level; PoV: probability of constraint violation; WS: worker skill; HLS: high-level skill; MLS: moderate-level skill; LLS: low-level skill; Scenario-1:33%HLS; 33%MLS;33%LLS ; Scenario-2: 66%HLS;33%LLS; Scneario-3:33%HLS;66%LLS.

Each scenario corresponds to a specific problem. Each problem is solved with the parameter Γ taking values 0 to 9. The PoV limit is set to 0.01 reflecting that only 1% chance the constraints can be violated. Besides, three different uncertainty coefficients (0.1, 0.2, and 0.5) are taking into account for each problem. Thus, total number of instances equals to 30.

Table 1 represents the results obtained for each instances. The parameter values are determined in accordance with the assembly line of a company producing water-meters.

To visualize the results given in Table 1, Figures 1, 2, and 3 are provided in this section.

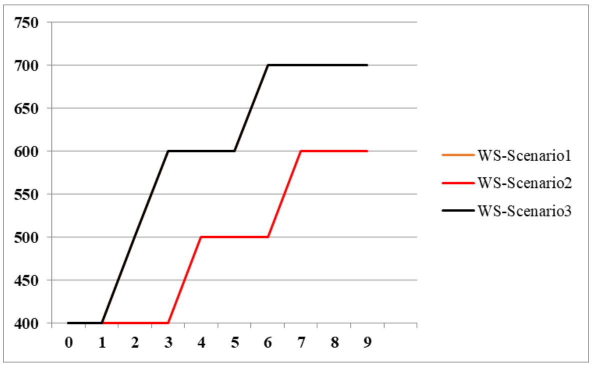

Fig. 1. Price of robustness for uncertainty level is 0.5

The price of robustness represents the value to protect a solution against the uncertainty with a pre-determined PoV value. Figure 1 represents the price of robustness value along with the change in the parameter Γ. According to this figure, the price of robustness is larger for scenario-3 in which 66% of total workers have low-skill levels.

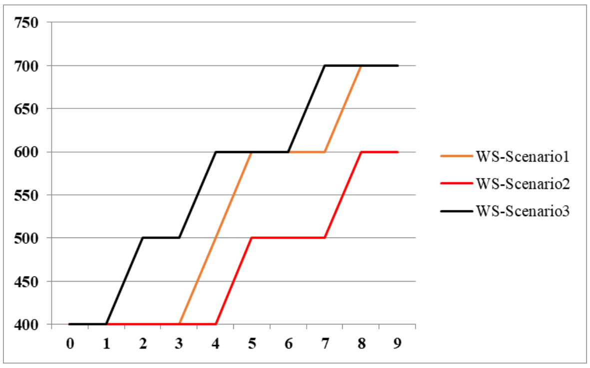

Fig. 2. Price of robustness for uncertainty level is 0.25

Figure 2 shows the change of price of robustness values along with the values of parameter Γ for 0.25 uncertainty level. The degree of uncertainty level determines how much the task times deviate. According to this figure, the best results are obtained when there are high-skill workers in the system. On the other hand, worse values are obtained for scenario-3 as in Figure 1.

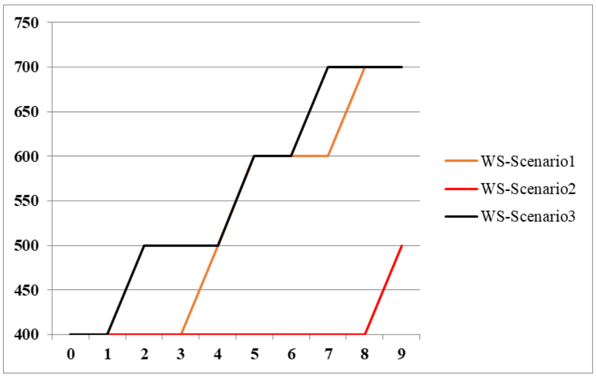

Fig. 3. Price of robustness for uncertainty level is 0.1

Figure 3 represents the price of robustness values along with the values of parameter Γ for 0.1 uncertainty level which is the lower than other levels. According to this figure, better results are obtained for scenario-2. It is interesting to state that PoV value equals 0.01 when the parameter Γ equals to 9. At this point, in Figure 3, the price of robustness has occurred for scenario 3, in other words, the system performance enhances as the number of high-skill workers increases in the system.

## IV. CONCLUSION

In this paper, the robust U-shaped assembly line balancing problem is examined with the task time uncertainty by considering the workers' heterogeneity. The robust optimization model is proposed for the addressed problem with the minimization objective of station utilization cost. To analyze the system performance in terms of the workers' proficiencies, three different scenarios are designed. Based on the scenarios, the uncertainty levels, and the robust parameters, 30 different problem instances are solved. The results are analyzed from the price of robustness perspective which is obtained for 0.01 PoV limit. The following managerial insights are revealed from the computational results.

When the results are examined with respect to the price of robustness, it is revealed that the best results are obtained for scenario-2 As the number of high-skill workers increases in the system, the resistance against the violation is higher.

The resistance against the violation reduces for scenario-3. Therefore, it is necessary to train the workers in the system to have robust solutions at lower prices.

As the degree of uncertainty (uncertainty level) decreases, better solutions are obtained especially for higher skill levels.

The solutions are obtained within reasonable computation time which means that the solutions are tractable.

Conservative solutions are achieved for less-skill levels.

Overall, it can be easily stated that the existence of high-skilled workers boosts system performance.

This study can be extended following directions: (i) different objective functions, such as operational cost, can be considered, (ii) other parameter uncertainties can be formulated, and (iii) last but not least, workers' proficiencies can be examined in detail.

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