

Modelling and Controller Design for Ball and Beam System

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Abstract – Ball and beam system is an experimental set for control system design that the position of the ball on a beam is changed with respect to the reference direction by changing the angle of the beam. The PID controller is applied to this problem. The performance of this controller is depended on the controller parameter values. On this study three optimization algorithms; Genetic Algorithm, Particle Swarm Optimization and Differential Evolution are applied to the tuning problem to get optimal parameters. The mathematical model of the system is evaluated by comparing the outputs from both real system and the simulation environment. In addition, the performance of these optimization algorithms with respect to the decision space boundaries is investigated by defining four difference ranges. The implementations are demonstrated on both the simulation environment and the ball and beam system hardware.

Keywords – ball and beam system; PID controller; implementation; optimization; genetic algorithm; particle swarm optimization; differential evolution.

I. INTRODUCTION

Ball and beam system (BBS) is a mechatronic system which is designed for the graduate and undergraduate control related studies [1,2] composed from a solid beam that is connected to a single joint, a ball on the beam, an actuator that changes the orientation of the beam, and related electronic and mechanical component. The aim of the efforts on this system is to change the position of the ball to reach it to a desired position. Since the ball is positioned on a beam that changes only on a single axis with respect to the beam, then this system can be referred as single degree of freedom (1-DoF) system. The position of the ball is changed by changing the angle of the beam from the actuator.

The controller design problem has been discussing by the researchers since 70s. However, in his paper only the papers published after 2000s are discussed for present the recent improvements for the BBS problem. In general, the relatively recent studies are focused on i) PID and PID-like controllers [3], ii) fractional-order controllers [4], iii) Fuzzy controllers [5] and iv) neuro controllers [6]. Even from these recent studies and their report related to the performance of the controller; it is observed that still PID controller is one of the major controllers for the BBS. The papers related to BBS is classified with respect to their focus as i) implementation: the BBS is used as the hardware tool and the performance of the controller algorithms is discussed from the output obtained from BBS hardware, ii) simulation: the mathematical model of BBS is programed on computer environment than controller performances are discussed; and iii) design: the mechanical and electronic configuration for BBS system is given; the construction of BBS is the main issue.

Implementation: As one of the PID controller implementation, the idea of improvement of the controller performance by using the switching among two controller algorithms is applied from neuro controller and PID

controllers is discussed in [7]. Even results present better performance, the settling time is relatively slow. It can be inferred that with a proper PID parameters, it is possible to get similar performance, even better. Also, PID-like algorithms are implemented on BBS system like I-PD that the integral term is subtracted from PD controller, and the performance is compared with PID controller [8]. The PID controller is implemented on Raspberry Pi hardware in [9]. The parameters are selected from Ziegler-Nichols method in the same paper. The filter at the feedback of the PID controller is investigated in [10]. It is show that the filter is improves the performance of the controller, hence it is also integrated on the system in this paper. The heuristic optimization algorithms are applied to get PD controller parameters where Bat Algorithm, Gravitational Search Algorithm and Firefly Algorithms are selected and applied to get parameters of the controllers [11].

Simulation: In [12], PID controller is applied on the BBS in simulation environment, and it is compared with Fuzzy controller. In [13], the on-line optimized PID controller is simulated. The controller is optimized with GA [14], and Particle Swarm Optimization in [15]. Even the performance of the simulation is in an acceptable level, the implementation of the proposed methodology does not look practice due to the implementation time of the optimization algorithm. Neuro Controller, LQR, P, and PID are applied to the system where Ziegler-Nichols is preferred as the tuning algorithm [16-19]. In [20], these parameters are obtained from trail-and-error. Ziegler Nichols and Tyreus-Luyben are compared on BBS in [21]. Almost same performance is reach for these two tuning algorithms but Tyreus-Luyben gives slightly better performance. Like conventional PID controller, also adaptive controller: Model reference Adaptive Controller is applied to the system [22].

Design: In general, all papers related to the design and construction of BBS is based on almost same structure so that the joint of the beam is either at one corner of the beam or at

the midpoint of the beam. However, the controller and position sensing differ at the papers as well as ball position sensor [3], FPGA [23], microprocessor [24], microcontroller [25], mechanical details [26], differs at each paper. After the investigation of these papers the following conclusions/lessons are obtained: i) Highly percentage of the papers are based on simulation environment, relatively few papers are published to give the results from the hardware, ii) The PID controller is one of the fundamental controller actions for the system, iii) Tuning of the PID controller is critical for the performance of the system.

In this research, initially the mathematical model of the system is presented with the aid of the similar studies. Then the structure of the applied system is investigated with respect to the impact of the actuator angle to the change of the beam angle. Next, the PID parameters are obtained from three optimization algorithms Genetic Algorithm, Particle Swarm Optimization, and Differential Evolution algorithms. To demonstrate the impact of the search space and the obtained local-optimal point, the implementations are repeated four different search space boundaries. All these parameters are implemented on both simulation environment and real-life BBS system [27, 28]. The performance of different controller parameters and the effect of the search space is discussed from the results.

This paper is organized as four more section after the first section "Introduction". In section 2 the mathematical model of the BBS is obtained. Section 3 presents the optimization algorithms. Section 4 is given for the implementation results and the last section is for the conclusion.

II. MODELLING

The mathematical model of the physical systems is the beginning point for controller design. The more accurate model makes the system design more accurate. Therefore, the definition of the physical system is one of the most important steps at the controller design.

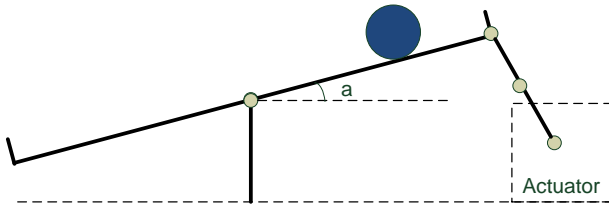


Fig. 1 The model of Ball and Beam System

The mathematical expressions are obtained by using the fundamental physical laws. For this purpose, two different methods, Lagrangian and Newton Methods can be applied to get the mathematical model. In this paper the mathematical model of the ball and beam system is expressed by using the Newton's Law of Motion.

$$\sum F = m_b \frac{d^2 x_b}{dt^2} \quad (1)$$

where m_b is the mass of ball, and x_b is the position of the with respect to the mid-point of the beam. All the forces on the ball is integrated to the system and the following nonlinear model is obtained. In the model J is the rotational inertia of the ball.

$$\ddot{x}_b = \frac{m_b g r_b^2}{m_b r_b^2 + J} \sin(a) - \frac{k r_b^2}{m_b r_b^2 + J} \dot{x}_b \quad (2)$$

Even this model presents the relation between the position of the ball and the angle of the beam, still the relation between

angle of the beam and angle of the actuator is needed. Therefore, from the relationship between the change of height is equal at the actuator side and the beam side than the additional relation is integrated on the model given in eq (2).

$$\ddot{x}_b = \frac{r_m m_b g r_b^2}{(m_b r_b^2 + J)L} \sin(a) - \frac{k r_b^2}{m_b r_b^2 + J} \dot{x}_b \quad (3)$$

where r_m is the length of the actuator arm and L is the half-length of the beam. The given nonlinear mathematical model is evaluated as the BBS at the simulation environment. Then the obtained controllers are implemented on a BBS hardware.

III. OPTIMIZATION ALGORITHMS

In this research, three optimization algorithms are selected as the tuning algorithms. These algorithms are Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Differential Evolution (DE) algorithms. At each of the algorithm, the mean of sum of the absolute error between reference point and the output is selected as the fitness -cost-function.

$$f = \frac{1}{N} \sum |R(i) - O(i)| \quad (4)$$

where O is the output of the system, R is the reference signal, and N is the number of time samples for each implementation. For each system the simulation time is restricted to 20 sec.

A. Genetic Algorithm

Genetic Algorithm (GA) is a heuristic optimization algorithm that is depended on the survival of the living beings at generations with genetic selection. From the optimization perspective, the algorithm is a population-based algorithm where each member is named as chromosomes. The first proposal of GA is depended on the binary representation of the chromosomes [29]. Hence it was more suitable for discrete optimization problems. The algorithm begins with the randomly assigned chromosomes. At each generation, every chromosomes are joint to form a new offspring with respect to their fitness -cost- value. This operator is called crossover. Then to alter the generation some digits of the chromosomes is changed. This is the mutation operator. Finally, among all the solutions, the best members survive to the next generation. In addition to solution some mechanisms are proposed, one of them is named as Elitism. This mechanism is based on to protect the best members and ensured to be survived to the next generation. As a summary, GA is based on three main operators: Crossover, Selection, and Mutation.

B. Particle Swarm Optimization

Particle Swarm Optimization (PSO) algorithm is a population-based nature-inspired optimization algorithm [30]. Unlike evolutionary optimization algorithms, the PSO is based on the movement of the animal flocks. The algorithm is a position-based algorithm so that each member holds a position and velocity. The position of the members is corresponding to the solution candidate. Each member records its best position and the best position among the swarm. Then it decides the velocity based on these landmarks. Hence, every member tends to move the best position of the swarm. As summary, the algorithm has three operators, record the best position so far, record the best position of the swarm and decide the next movement.

Table 1. Time Response Properties of Each Implementation

[x,y,z]	[2, 1, 1]						[3, 1, 1]					
	Simulation			Hardware			Simulation			Hardware		
Algorithm:	GA	PSO	DE	GA	PSO	DE	GA	PSO	DE	GA	PSO	DE
Rise Time:	0,8870	0,9013	0,9142	0,6408	0,7415	0,7361	0,5882	0,7668	0,7356	0,4112	0,7436	0,6855
Overshoot:	53,70	53,85	54	70,5	66,5	69,3	87,94	39,57	39,83	145,70	45,1	57
Peak Time:	3,9544	4,0803	4,1057	2,8287	2,8765	2,8207	2,103	4,1682	3,83	2,0598	2,49	2,3426
S. Error:	1,2561	1,2504	1,2604	1,97	2,16	3,14	0,8802	0,8404	0,8444	4,87	1,76	1,85
[x,y,z]	[5, 1, 1]						[2, 2, 2]					
	Simulation			Hardware			Simulation			Hardware		
Rise Time:	1,2685	1,2693	1,2690	1,1885	1,1916	1,1669	0,9669	0,8999	0,8961	0,7326	0,7015	0,7934
Overshoot:	28,27	28,75	28,6053	21,9	23,70	20,8	54,96	53,80	54,09	59,9	66,5	65,3
Peak Time:	6,6677	6,708	6,7076	5,85	5,38	4,3904	4,2549	4,066	4,100	3,1928	2,8406	2,8964
S. Error:	0,4884	0,482	0,4866	1,72	0,96	1,78	1,3115	1,2482	1,2537	2,4	2,31	2,23

C. Differential Evolution

Differential Evolution (DE) is a population based evolutionary algorithm; like GA it is based on the same operators [31]. Like GA, it contains three operators however the order of these operators and the methodology at the operators differs from GA. In DE, the algorithm begins with the mutation which is an update rule that changes the member value. Then, crossover and selection operators are applied to the population. At crossover, the offspring are formed from a random variable as the decision. Finally, the best members among the parents and offspring are survived to the next generation.

IV. IMPLEMENTATION, RESULTS, AND DISCUSSION

In this research, the feedback PID controller is applied to BBS. The controller structure in frequency domain is given as below.

$$C(s) = K \left(1 + T_i \frac{1}{s} + T_d s \right) \tag{5}$$

The PID controller contains three parameters K , T_i , and T_d . The controller parameters are tuned by using three optimization algorithms from simulation environment. The nonlinear mathematical model of the BBS is controlled with an PID controller. However, in real life the actuator and connection between actuator and control action should be modeled for a better performance. For this purpose, a deterministic structure is applied for a servo system with respect to the real system. Figure 2 gives the Acrome-BBS [27,28] that is the hardware for the controller design.



Fig. 2 The Ball and Beam System, Acrome.

The optimization algorithms search the -local- optimal point inside the given search space. Therefore, to get the global optimum point the correct boundaries for the search space should be defined. Therefore, in this study, four different search space boundaries are defined for three-dimensional search space. These are i) [2,1,1], ii) [3,1,1], iii) [5,1,1], and iv) [2,2,2]. It is desired to get the best possible solution among the boundaries. Table 1 and Table 2 are the obtained result for the BBS.

Table 2. PID Controller Parameters

[x,y,z]	[2,1,1]			[3,1,1]		
	GA	PSO	DE	GA	PSO	DE
K	1,9873	1,9962	1,9803	2,8430	2,9830	2,9536
T_d	0,7549	0,7888	0,8090	4,8710e-4	0,6645	0,6070
T_i	0,7511	0,9831	0,9909	0,7200	0,9976	0,6661
[x,y,z]	[5,1,1]			[2,2,2]		
	GA	PSO	DE	GA	PSO	DE
K	4,9988	5	4,9938	1,9031	1,9998	1,9910
T_d	0,9987	1	0,9999	0,8879	0,7867	0,7806
T_i	0,4008	0,7692	0,4706	1,0420	0,9813	1,0064

Table 1 gives the numerical values for both simulation and experimental results and Table 2 gives the related PID parameters with respect to the different boundaries. From Table 1 it can be inferred from the results that, i) the boundaries influence the performance of all algorithms. It is not possible to get the same performance for a larger boundary, ii) for all cases the steady state error remains, where the minimum steady state error is 0.48 for simulation and 0.9 for implementation, iii) larger boundary [5,1,1] gives the best performance among other ranges, iv) from overall results, the best performance is obtained from DE algorithm. That demonstrates the impact of the optimization algorithm on the problem, and v) for all cases the overshoot is obtained, however the minimum overshoot is almost 20% that corresponding to approximately 2 cm. Figure 3 shows the obtained time response properties in Table 1 for simulation and Figure 4 gives the implementations results given in Table 1. It is also observed for these figures that the performance of algorithms is like simulation and implementations.

V. CONCLUSION

In this research, the optimal PID controller is obtained by using three optimization algorithms. These controllers are applied to simulation environment and hardware. These results obtained from simulation and hardware system are compared from both numerical values and graphical output signals. It is observed that the mathematical model and implementation results are very similar to each other. For the different set of PID parameters, simulation and implementation results can point almost the same performance. From the result, it is showed that the optimization algorithms and the search space boundaries have an impact on the performance. Even the time response of the system is in an acceptable level, the steady state error is larger than the acceptable level of 0.05 or 0.02. As the future study, an additional sensor will be installed on the BBS, and different controller algorithms -like Fuzzy- will be implemented and optimized.

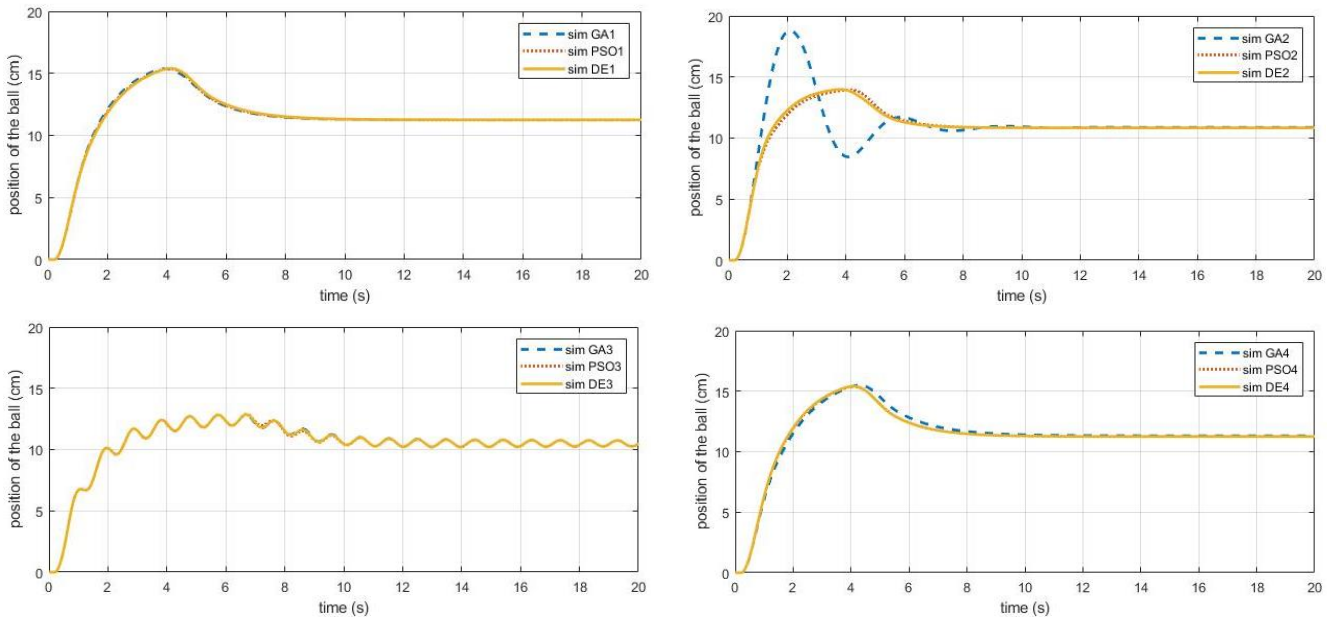


Fig. 3 Simulation results for [2,1,1] (upper left), [3,1,1] (upper right), [5,1,1] (bottom left), and [2,2,2] (bottom right).

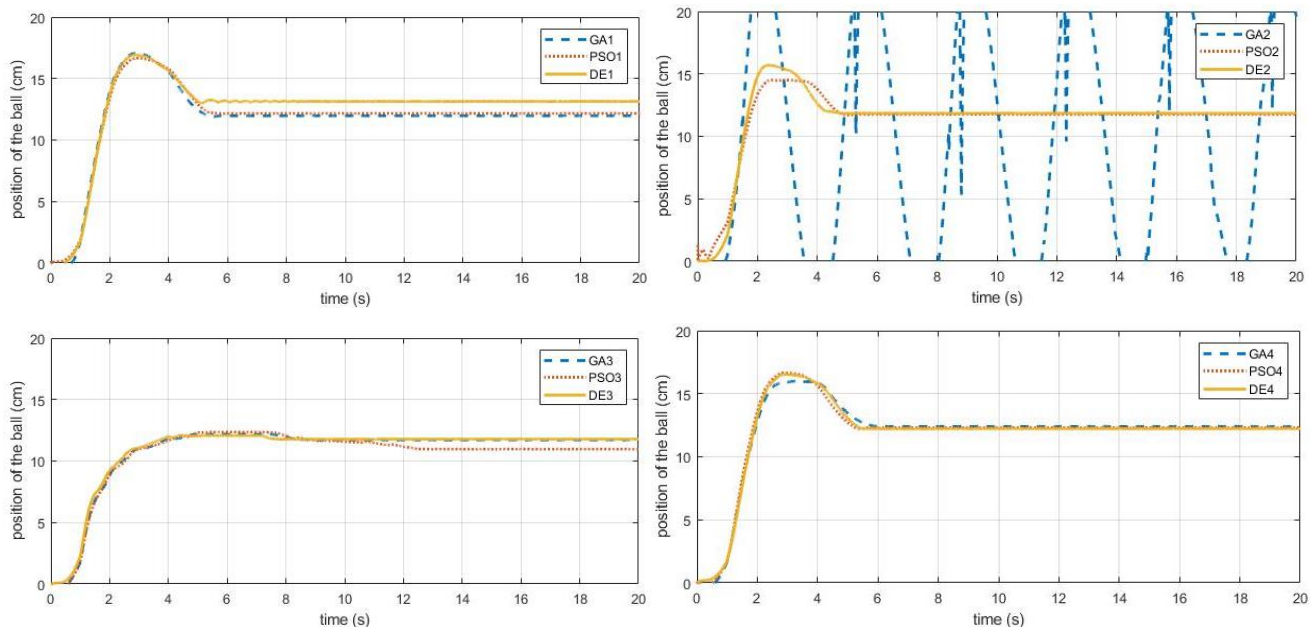


Fig. 4 Implementation results for [2,1,1] (upper left), [3,1,1] (upper right), [5,1,1] (bottom left), and [2,2,2] (bottom right).

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